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higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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AUGUST EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 August 2014 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $w = \tan 3x \,.\, \cot \dfrac{y}{3}$, determine $\dfrac{\partial^2 w}{\partial x^2}$ (2)

1.2 The parametric equations of a function are given as:

$y = \dfrac{3a}{1+a^2}$ and $x = \dfrac{4}{1+a^2}$, calculate the values of the following:

1.2.1 $\dfrac{dy}{dx}$

1.2.2 $\dfrac{d^2y}{dx^2}$

(2 × 2) (4)
[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \tan^4 2x$ (4)

2.2 $y = \sqrt{12 - 8x - 2x^2}$ (4)

2.3 $y = \dfrac{\sin^3 \frac{x}{3} \,.\, \sec^2 \frac{x}{3}}{1 + \tan^2 \frac{x}{3}}$ (4)

2.4 $y = 3x^2 \,.\, \cot^{-1} x$ (4)

2.5 $y = x \ln ax$ (2)
[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{4+3x-x^2}{x(3-x)^2}\,dx$ (6)

3.2 $\int \frac{x^2+6x+12}{x(2-x^2)}\,dx$ (6)

[12]

QUESTION 4

4.1 Find the equation of the curve with gradient $\frac{dy}{dx}=x^2-\frac{2y}{x}$ and which passes through (1;1). (5)

4.2 Calculate the particular solution of:

$$\frac{d^2s}{dt^2}-9s=e^t,\ \text{if } t=0 \text{ when } s=1 \text{ and } \frac{ds}{dt}=3 \text{ when } t=0.$$

(7)

[12]

QUESTION 5

5.1 5.1.1 Sketch of the curve of $y = -x^2+3x$ and show the representative strip/element that you will use to calculate the volume (by using the SHELL-method only) generated when the area bounded by the curve, $x=0$ and the line $y=3$ is rotated about the y-axis. (2)

5.1.2 Use the shell-method to calculate the volume described in QUESTION 5.1.1. (4)

5.2 5.2.1 Make a neat sketch of the graph $x^2-y^2=1$ and show the area bounded by the graph and $x=2$. Show the representative strip/element that you will use to calculate the bounded area. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (4)

5.2.3 Calculate the x-ordinate of the centroid of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Calculate the points of intersection of the graphs $y = 3x$ and $y = 3x^2$. Make a neat sketch of the two graphs and show the representative strip/element (Parallel to the y -axis) that you will use to calculate the volume if the area bounded by the two graphs is rotated about the y -axis. (3)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (4)

5.3.3 Calculate the moment of inertia of the solid obtained when the area in QUESTION 5.3.1 is rotated about the y -axis and express the answer in terms of the mass. (5)

5.4 5.4.1 A vertical weir in a rectangular canal is 6 m wide and 3 m high. The top of the weir is 3 m below the water surface. Make a neat sketch of the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the weir. (2)

5.4.2 Calculate the area moment of the weir about the water surface by means of integration. (4)

5.4.3 Calculate the second moment of area of the weir about the water surface, as well as the depth of the centre of pressure on the weir by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve given by the parametric equations

$x = 3(\cos\theta + \theta\sin\theta)$ and $y = 3(\sin\theta - \theta\cos\theta)$ between $\theta = 0$ and $\theta = \dfrac{\pi}{2}$. (6)

6.2 Determine, by integration, the surface area generated when revolving $\dfrac{x^2}{36} + \dfrac{y^2}{4} = 1$, about the x -axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}\,f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\,\frac{d}{dx}\,x^n$	$a \int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}\,(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}\,(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}\,ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\,\frac{d}{dx}\,f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}\,f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}\,f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln\left[\sec(ax)\right]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln\left[\sin(ax)\right]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln\left[\sec ax+\tan ax\right]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\text{cosec} f(x)$	$-\text{cosec} f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx \; ; \; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy \; ; \; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \, ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \, ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \, ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \, ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \, ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$