higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N5

22 NOVEMBER 2016

This marking guideline consists of 9 pages.

INSTRUCTIONS AND INFORMATION

1. ✓ *equals* 1 *mark*.

2. Half marks are not allocated, unless indicated otherwise.

3. Using the wrong formula when one is required is a principle error and NO marks should be allocated.

4. Candidates should show ALL formula and intermediate steps and simplify where possible.

5. ALL final answers must be rounded off to THREE decimal places (unless indicated otherwise).

6. Questions may be answered in any order, but subsections of questions must be kept together otherwise candidates can be penalised with ONE mark.

7. When something has been copied incorrectly from the question paper and the standard of the question is still the same, the candidate will be penalised by ONE mark. If the copying error simplifies the question the mark will be forfeited.

8. Questions must be answered in blue or black ink. Answers in PENCIL will not be marked as it is regarded as rough work.

QUESTION 1

1.1 1.1.1

$$\lim_{x\to\infty} -\frac{x}{\ln x} - \frac{1}{\ln x}$$

$$\lim_{x\to\infty} \frac{x-1}{\ln x} \quad ✓$$

$$= \lim_{\to\infty} \frac{1}{\frac{1}{x}} \quad ✓$$

$$= \infty \quad ✓$$

(3)

1.1.2

$$\lim_{x\to 0} \frac{x^3 - 2x - 5}{x^2 - 2x}$$

$$\frac{-5}{0} \quad ✓$$

$$= \infty \quad ✓$$

(2)

1.2

$$f(-1) = \sqrt{\frac{-9}{2}} \quad ✓$$

undefined

discontinuous ✓

(2)
[7]

QUESTION 2

2.1 $f(x) = \sqrt[3]{2x}$

✓✓ *binomial theorem*

$$f'(x) = \lim_{h\to 0} \frac{\sqrt[3]{2}(x^{\frac{1}{3}} + \frac{1}{3}x^{\frac{-2}{3}}h - \frac{2}{9}x^{\frac{-5}{3}}\frac{h^2}{2} + \ldots) - \sqrt[3]{2}x^{\frac{1}{3}}}{h} \quad ✓$$

$$= \lim_{h\to 0} \frac{h(\frac{1}{3}\sqrt[3]{2}x^{\frac{-2}{3}} - \frac{2}{9}\sqrt[3]{2}x^{\frac{-5}{3}}\frac{h}{2} + \ldots)}{h} \quad ✓$$

$$= \frac{\sqrt[3]{2}}{3x^{\frac{2}{3}}} \quad ✓$$

(5)

2.2 2.2.1

$$y = \ln^2 \frac{3x}{e^x} - \ln 3x^2$$

$$\frac{dy}{dx} = 2(\ln \frac{3x}{e^x}).\frac{1}{\frac{3x}{e^x}}.\frac{3e^x - 3x.e^x}{e^{2x}} - \frac{6x}{3x^2}$$

(4)

2.2.2 $y = 5\arccos ec(x\sin 2x)$

$$y' = \frac{5(1\sin 2x + 2x\cos 2x)}{x\sin 2x\sqrt{(x\sin 2x)^2 - 1}}$$ ✓✓ product rule ✓

(3)

2.2.3 $y = 4\sqrt{\cos^3(x^2 - x - 1)}$

$$y' = 4.\frac{3}{2}\cos^{\frac{1}{2}}(x^2 - x - 1).-\sin(x^2 - x - 1)(2x - 1)$$

(3)

2.3 $s(t) = t^{\cos t}$

$\ln s = \cos t.\ln t$ ✓

$\frac{1}{s}\frac{ds}{dt} = -\sin t.\ln t + \frac{\cos t}{t}$ ✓

$\frac{ds}{dt} = s(-\sin t.\ln t + \frac{\cos t}{t})$ ✓

$for\ \ t = \frac{\pi}{4} : s = 0{,}843$

$velocity = 0{,}843(0{,}171 + 0{,}9)$ ✓

$= 0{,}903$ ✓

(5)

2.4 $\cos(x + y) + \sin x + \sin y = 2$

$-\sin(x + y).(1 + \frac{dy}{dx}) + \cos x + \cos y.\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-\sin(x + y) + \cos y) = -\cos y + \sin(x + y)$ ✓

$\frac{dy}{dx} = \frac{-\cos y + \sin(x + y)}{-\sin(x + y) + \cos x}$

$\frac{dy}{dx} = -1$ ✓

(4)
[24]

QUESTION 3

3.1 3.1.1 $f(x) = 3x^3 - 4x^2 - 2x + 2$

$9x^2 - 8x - 2 = 0$ ✓

$x = 1{,}092 \quad \text{of} \quad x = -0{,}203$

$y = -1{,}047 \text{ of } y = 2{,}216$

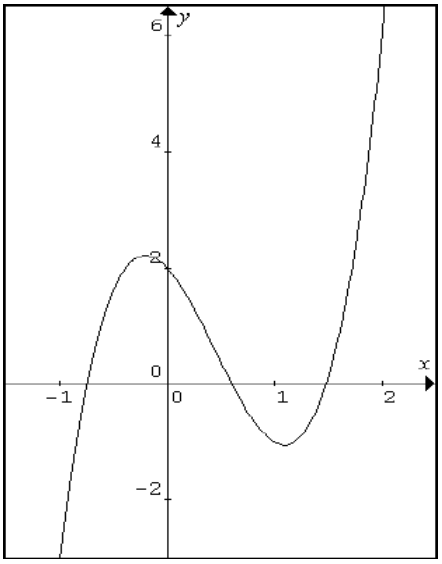
✓for both x-values
✓ for both y-values (3)

3.1.2 $0 = 3x^3 - 4x^2 - 2x + 2 \ : \ -1 \le x \le 2$

x-1	0	1	2✓
y	3	2	-1	6✓

(2)

3.1.3



✓ y-intercept
✓ turning points
✓ shape (3)

3.1.4 $e = -\dfrac{1{,}779}{10{,}41} = -0{,}1709$ ✓

$r = 1{,}7 - 0{,}1709 = 1{,}5291$ ✓ (2)

3.2 3.2.1 $A = xy$ ✓

$C = 2(36x) + 2(45y)$

$C = 72x + 90y$ ✓ (2)

3.2.2 $56000 = 72x + 90y$

$\dfrac{56000 - 72x}{90} = y$ ✓

$A = 622{,}22x - 0{,}8x^2$ ✓

$622{,}22 - 1{,}6x = 0$ ✓

$388{,}89 = x$ ✓

$311{,}11 = y$ ✓ (5)

3.3 $V = \pi.r^2.h$

$r = 1{,}5m$

$\frac{dV}{dt} = \pi(r)^2.\frac{dh}{dt}$ ✓ derivative

$3 \checkmark = \pi(1{,}5)^2 \checkmark \frac{dh}{dt}$ substitution

$0{,}424m = \frac{dh}{dt}$ ✓

(4)
[21]

QUESTION 4

4.1 $\int \frac{1-\cos 2x}{-2x+\sin 2x}dx$ $\frac{d}{dx}(-2x+\sin 2x) = -2+2\cos 2x$

$\frac{1}{-2}\int \frac{-2(1-\cos 2x)}{-2x+\sin 2x}dx$ ✓

$= \frac{-1}{2}\ln(-2x+\sin 2x)+c$ ✓

(2)

4.2 4.2.1 $\int 2x.e^{2x}dx$

$2x.\frac{e^{2x}}{2} \checkmark - \int 2e^{2x} \checkmark dx$

$= xe^{2x} - e^{2x} + c$ ✓ (3)

4.2.2 $\int \frac{2x^3-4}{x+1}dx$

Long division:

✓✓quotient remainder✓

$\int 2x^2 - 2x + 2 + \frac{-6}{x+1}dx$

$= \frac{2x^3}{3} - x^2 + 2x - 6\ln(x+1) + c$ ✓✓ ONE mark for two correct terms

(5)

4.2.3 $\int \frac{\sin^3 3x}{\sin 3x} dx$

$\int \sin^2 3x dx$

$= \int \frac{1}{2} - \frac{1}{2}\cos 6x dx$ ✓

$= \frac{1}{2}x - \frac{\sin 6x}{12} + c$ ✓✓

(3)

4.2.4 $\int 5x^2\sqrt{2x^3+1}\,dx$

$u = 2x^3 + 1$

$dx = \frac{du}{6x^2}$ ✓

$\int 5x^2 . u^{\frac{1}{2}} . \frac{du}{6x^2} dx$

$= \frac{5}{6} . \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$ ✓

$= \frac{5}{9}(2x^3+1)^{\frac{3}{2}} + c$ ✓

(3)

4.3 $\int \frac{4x^2 - 3x + 1}{4x^2 - 1}$

$\frac{4x^2 - 3x + 1}{(2x-1)(2x+1)} = \frac{A}{2x-1} + \frac{B}{2x+1}$ ✓

$4x^2 - 3x + 1 = A(2x+1) + B(2x-1)$ ✓

$x = -0{,}5 : -\frac{7}{4} = B$ ✓

$x = 0{,}5 : \frac{1}{4} = A$

$\int \frac{\frac{1}{4}}{(2x-1)} - \frac{\frac{7}{4}}{2x+1} dx$ ✓

$= \frac{1}{8}\ln(2x-1) - \frac{7}{8}\ln(2x+1) + c$ ✓

(5)

4.4 $\int \sin(\arcsin x^{-1})dx$

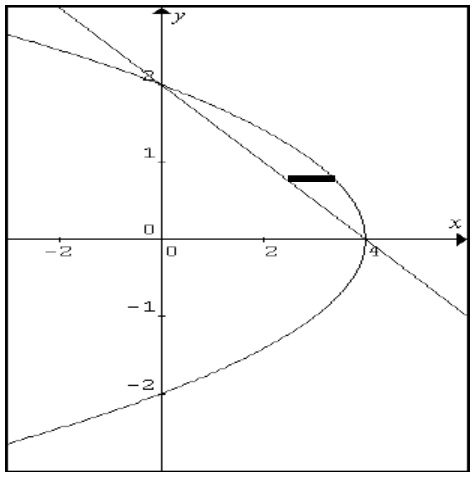
$\int x^{-1}dx$ ✓

$= \ln x + c$ ✓

(2)
[23]

QUESTION 5

5.1 5.1.1 $x = -y^2 + 4$ and $x = 4 - 2y$



✓ - parabola graph
✓ - straight line
✓ - shading and strip

(3)

5.1.2 $\int_0^2 (-y^2 + 4) - (4 - 2y)dy$ ✓- equations

$= [\frac{-y^3}{3} + y^2]_0^2$ ✓ ✓ - integration

$= 1{,}333$ ✓

(4)

5.1.3 $V = \pi \int_0^2 (-y^2 + 4)^2 - (4 - 2y)^2 dy$ ✓ - equations

$= \pi \int_0^2 y^4 - 12y^2 + 16y dy$ ✓- simplification

$= \pi [\frac{y^5}{5} - 4y^3 + 8y^2]_0^2$ ✓- integration

$= \pi(6{,}4)$

$= 20{,}106$ ✓

(4)

5.2

$$2\int_0^{\frac{a}{2}} x^2 . \frac{m}{ab} . b dx \quad \checkmark$$

$$= 2\frac{m}{a}\int_0^{\frac{a}{2}} x^2 dx$$

$$= 2\frac{m}{a}[\frac{x^3}{3}]_0^{\frac{a}{2}} \quad \checkmark$$

$$= \frac{2m}{a}(\frac{a^3}{24} - 0) \quad \checkmark$$

$$= \frac{ma^2}{12} \quad \checkmark$$

(4)
[15]

QUESTION 6

6.1

$$e^{2x+3y} . \frac{dx}{dy} = e^{-5x+8y}$$

$$\frac{e^{2x}}{e^{-5x}} dx = \frac{e^{8y}}{e^{3y}} dy$$

$$e^{7x} dx = e^{5y} dy \quad \checkmark \quad \checkmark$$

$$\frac{e^{7x}}{7} = \frac{e^{5y}}{5} + c \quad \checkmark \quad \checkmark$$

(4)

6.2

$$\frac{d^2y}{dx^2} = 2.10^{4x} + \frac{1}{\sin^2 2x} - 4$$

$$\frac{dy}{dx} = \frac{2.10^{4x}}{4\ln 10} - \frac{\cot 2x}{2} - 4x + c \quad \checkmark \quad \checkmark \quad \checkmark$$

$$y = \frac{10^{4x}}{2.4\ln^2 10} - \frac{\ln(\sin 2x)}{4} - 2x^2 + cx + d \quad \checkmark \quad \checkmark \quad \checkmark$$

(6)
[10]

TOTAL: 100