higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N3

1 APRIL 2016

This marking guideline consists of 10 pages.

QUESTION 1

1.1 1.1.1

$$x(3x-2)-y(3y+2)$$
$$=3x^2-2x-3y^2-2y \quad ✓$$
$$=3(x^2-y^2)-2(x+y) \quad ✓$$
$$=3(x-y)(x+y)-2(x+y)$$
$$=(x+y)[3(x-y)-2]$$
$$=(x+y)[3x-3y-2] \quad ✓ \; ✓$$

(4)

1.1.2

$$4n^{4p}+3n^{2p}-1$$
$$=(n^{2p}+1)(4n^{2p}-1) \quad ✓ \; ✓$$

(2)

1.2

$$f(x)=2x^3+x^2-5x+2$$
$$\therefore f(1)=2(1)^3+(1)^2-5(1)+2$$
$$=2+1-5+2 \quad ✓$$
$$=0$$

$\therefore$ x-1 is a factor of f(x) ✓

$$\begin{array}{r} 2x^2+3x-2 \\ x\text{-}1 \;\overline{)\;2x^3+x^2-5x+2} \\ \underline{2x^3-2x^2} \\ 3x^2-5x \\ \underline{3x^2-3x} \\ -2x+2 \\ \underline{-2x+2} \\ \cdot \quad \cdot \end{array} \quad ✓$$

$$\therefore f(x)=(x-1)(2x^2+3x-2)$$
$$=(x-1)(2x-1)(x+2) \quad ✓ \; ✓$$

(5)

1.3 $\dfrac{x-1}{x+1}-\dfrac{2x-1}{3-x}+\dfrac{2x^2-7x-17}{x^2-2x-3}$

$=\dfrac{x-1}{x+1}+\dfrac{2x-1}{x-3}+\dfrac{2x^2-7x-17}{(x+1)(x-3)}$ ✓✓

$=\dfrac{(x-1)(x-3)+(2x-1)(x+1)+2x^2-7x-17}{(x+1)(x-3)}$ ✓

$=\dfrac{x^2-4x+3+2x^2+x-1+2x^2-7x-17}{(x+1)(x-3)}$ ✓

$=\dfrac{5x^2-10x-15}{(x+1)(x-3)}$

$=\dfrac{5(x^2-2x-3)}{(x+1)(x-3)}$ ✓

$=\dfrac{5(x+1)(x-3)}{(x+1)(x-3)}$

$=5$ ✓

(6)
[17]

QUESTION 2

2.1
$$\frac{\sqrt{x}-\frac{1}{2\sqrt{x}}}{3x^{\frac{3}{2}}}$$
$$=\frac{2x-1}{2x^{\frac{1}{2}}}\div 3x^{\frac{3}{2}} \quad \checkmark$$
$$=\frac{2x-1}{2x^{\frac{1}{2}}}\times\frac{1}{3x^{\frac{3}{2}}} \quad \checkmark$$
$$=\frac{2x-1}{6x^2} \quad \checkmark \quad \checkmark$$
(4)

2.2
$$\log_{0,5}128$$
$$=\frac{\log_2 128}{\log_2\frac{1}{2}} \quad \checkmark \quad \checkmark$$
$$=\frac{\log_2 2^7}{\log_2 2^{-1}} \quad \checkmark$$
$$=\frac{7\log_2 2}{-1\log_2 2} \quad \checkmark$$
$$=-7$$
(4)

2.3 2.3.1
$$\sqrt{16+3^{2x}}=3^x+2$$
$$\therefore 16+3^{2x}=\left(3^x+2\right)^2 \quad \checkmark$$
$$\therefore 16+3^{2x}=3^{2x}+4.3^x+4 \quad \checkmark$$
$$\therefore 12=4.3^x \quad \checkmark$$
$$\therefore 3^x=3 \quad \checkmark$$
$$\therefore x=1$$
TEST : If $x=1$ then LHS=RHS=5
(4)

2.3.2
$$(\log x-2)\times\log(x-2)=0$$
$$\therefore \log x-2=0 \quad \text{or} \quad \log(x-2)=0$$
$$\therefore \log x=2 \ \checkmark \qquad x-2=10^0 \ \checkmark$$
$$\therefore x=10^2 \qquad x-2=1$$
$$\therefore x=100 \ \checkmark \qquad x=3 \ \checkmark$$
(4)

[16]

QUESTION 3

3.1 $4x = 48 - 2x^2$

$2x^2 + 4x = 48$✓

$x^2 + 2x = 24$

$x^2 + 2x + 1 = 24 + 1$✓

$(x+1)^2 = 25$

$x + 1 = \pm\sqrt{25}$✓

$x = -1 \pm 5$

$x = -6 \quad or \quad x = 4$✓ (4)

3.2 $D = \sqrt{\dfrac{x+b}{x-b}}$

$D^2 = \dfrac{x+b}{x-b}$ ✓

$D^2x - D^2b = x + b$✓

$D^2b + b = D^2x - x$

$b(D^2 + 1) = D^2x - x$✓

$b = \dfrac{D^2x - x}{D^2 + 1} = \dfrac{(D^2 - 1)x}{D^2 + 1}$ ✓

(4)

3.3 $\log_e t - \log_e p + \log_e w = ds$

$\log_e \frac{tw}{p} = ds$ ✓

$\frac{tw}{p} = e^{ds}$ ✓

$w = \frac{p}{t} e^{ds}$ ✓ (3)

3.4 Deposit is R$3x$ ✓
Total price is R$33x$
Total amount for 9 instalments= R$30x$ ✓
$\therefore$ Each monthly instalment $= \text{R}\frac{30x}{9} = R\frac{10x}{3}$ ✓ ✓ (4)
[15]

QUESTION 4

4.1 4.1.1 $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (5-1)^2}$

$= \sqrt{4+16}$ ✓

$= \sqrt{20}$

$= 2\sqrt{5}$ ✓ (2)

4.1.2 $BC = 2\sqrt{5}$

$\therefore \sqrt{(0-4)^2 + (k-5)^2} = \sqrt{20}$

$\therefore 16 + (k-5)^2 = 20$ ✓

$\therefore (k-5)^2 = 4$

$\therefore k - 5 = \pm 2$ ✓

$\therefore k - 5 = 2 \quad or \quad k - 5 = -2$

$\therefore k = 7$ ✓ $\quad k = 3$ ✓ (4)

4.1.3 $\therefore M_{AB} = \frac{5-1}{4-2} = 2$ ✓

$\therefore M_{CB} = \frac{7-5}{0-4} = -\frac{1}{2}$ ✓

$M_{AB} \times M_{CB} = -1$

therefore CB ⊥ AB ✓ (3)

4.1.4 Area of $\Delta ABC = \frac{1}{2} \times$ base × height

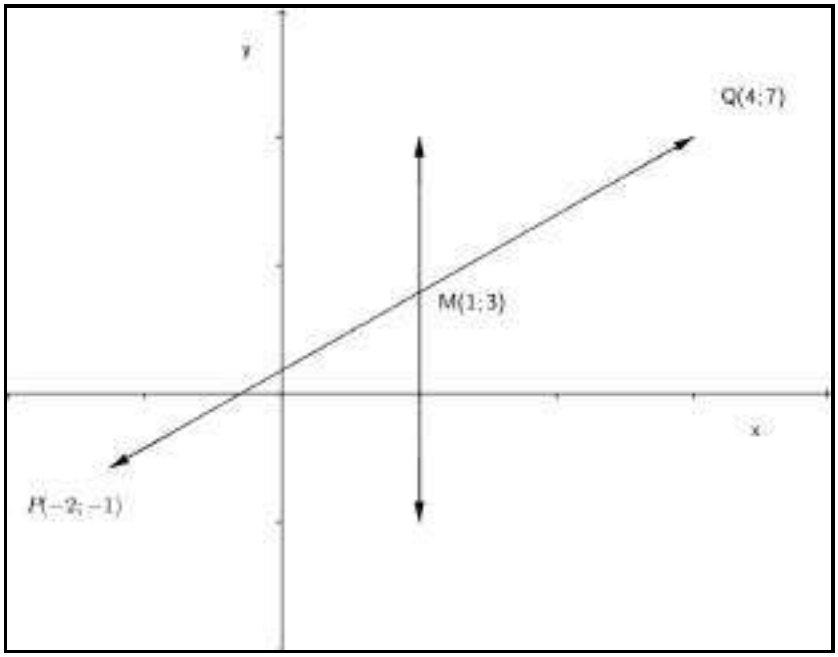
$= \frac{1}{2} AB \times BC$ ✓

$= \frac{1}{2}\sqrt{20} \times \sqrt{20}$ ✓

$= 10$ units^2 ✓

(3)

4.2



$$\text{coordinates of M} = \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+4}{2};\frac{-1+7}{2}\right)$$

$=(1;3)$ ✓ ✓

$\therefore$ Equation $x=1$ ✓ ✓

(4)

4.3 $M_{AB}=1$ ✓ $M_{AC}=3$ ✓

$\therefore \tan\theta = 1$ $\therefore \tan\alpha = 3$

$\therefore \theta = 45^\circ$ ✓ $\therefore \alpha = 71{,}565^\circ$ ✓

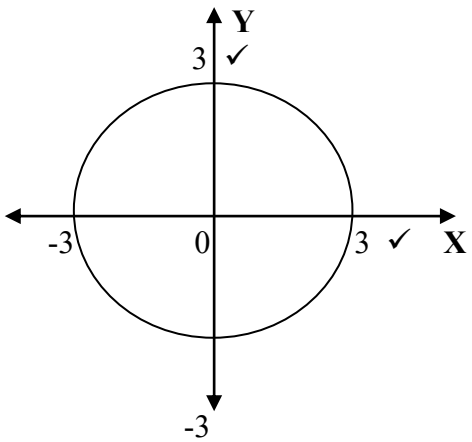
$\hat{BAC} = \alpha - \theta = 71{,}565^\circ - 45^\circ = 26{,}565^\circ$ ✓

(5)

[21]

QUESTION 5

5.1 $3x^2 + 3y^2 = 27$



(2)

5.2.1 $y = x^3 + 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 + 12x + 9$ ✓

Let $y = 0$

$\therefore \ + 3x^2 + 12x + 9 = 0$

$3(x^2 + 4x + 3) = 0$

$(x+1)(x+3) = 0$ ✓ ✓

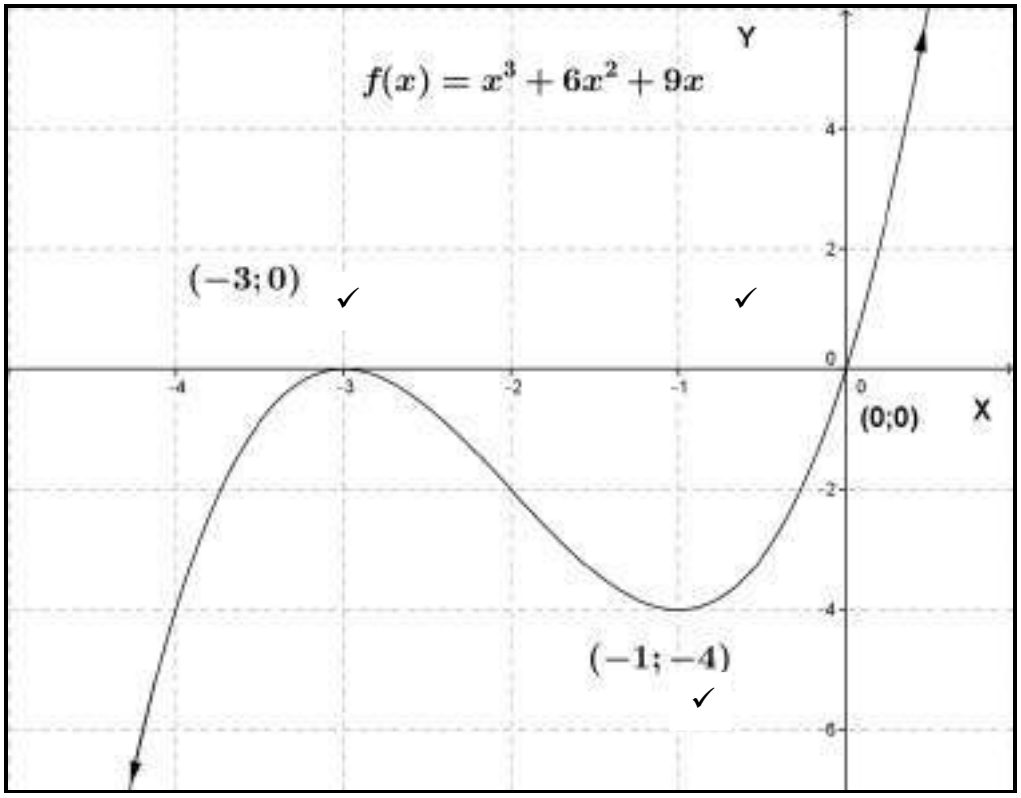
$x = -1$ or $x = -3$

$f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) = 4$

$f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) = 0$ ✓ ✓

Turningpoints $(-3;0)$ and $(-1;4)$ (5)

5.2.2



✓ ✓ ✓ (3)

5.3 $y = \frac{1}{x} + 2\sqrt{x}$

$y = x^{-1} + 2x^{\frac{1}{2}}$ ✓

$\frac{dy}{dx} = -x^{-2} + 2(0,5)x^{-\frac{1}{2}}$ ✓

$\frac{dy}{dx} = -\frac{1}{x^2} + \frac{1}{\sqrt{x}}$ ✓ ✓ (4)

[14]

QUESTION 6

6.1 $LHS = sin^2A + tan^2A + cos^2A$

$= (sin^2A + cos^2A) + \tan^2 A$ ✓

$= 1 + \tan^2 A$ ✓✓

$= sec^2A = RHS$ ✓ (4)

6.2 $\sin\theta = 1 - \cos^2\theta$

$\therefore \sin\theta = \sin^2\theta$ ✓

$\therefore \sin^2\theta - \sin\theta = 0$

$\therefore \sin\theta(\sin\theta - 1) = 0$ ✓

$\therefore \sin\theta = 0$ or $\sin\theta = 1$

$\therefore \theta = 0^\circ$ or $\theta = 90^\circ$ ✓

or $\theta = 180^\circ$ ✓ ✓ (5)

6.3 6.3.1

In ΔABC:

$\frac{AB}{\sin 70^\circ} = \frac{30}{\sin 45^\circ}$ ✓

$\therefore AB = \frac{30\sin 70^\circ}{\sin 45^\circ}$ ✓

$= 39,868m$ ✓ (3)

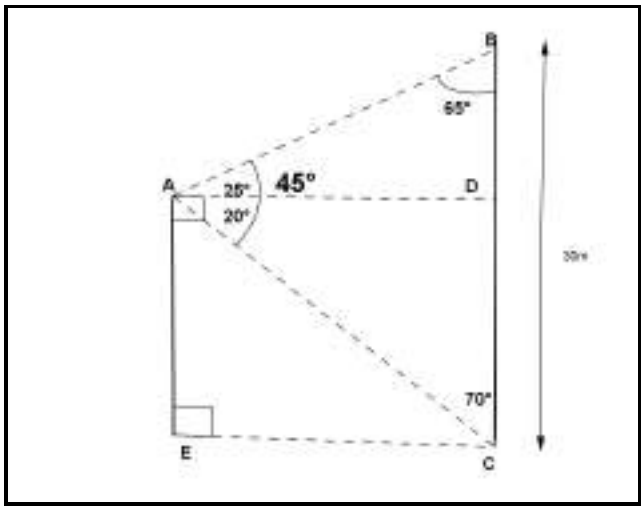
6.3.2 *In* ΔABD:

$\frac{AD}{AB} = \cos 25^\circ$ ✓

$\therefore AD = AB\cos 25^\circ$ ✓

$= 39,868 \times \cos 25^\circ$

$= 36,133m$ ✓ (3)



6.4 $a = 2$ ✓

$p = 2$ ✓ (2)

[17]

TOTAL: 100