DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

APRIL 2013

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before you answer the questions.

1. Answer all the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL the calculations and intermediary steps.
5. Questions may be answered in any order but subsections of questions must NOT be separated.
6. Round off ALL answers to THREE decimal places.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. ALL graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied.
9. Diagrams are NOT drawn to scale.
10. A formula sheet is included at the end of this question paper. The list is NOT necessarily complete. Any other applicable formula may be used.
11. Write neatly and legibly.

NON-PROGRAMMABLE AND NON-GRAPHICAL CALCULATORS MAY BE USED

QUESTION 1

1.1 Indicate whether the following statements are TRUE or FALSE. Write only 'true' or 'false' next to the question number (1.1.1–1.1.3) in the ANSWER BOOK

1.1.1 $\log\frac{a}{b}=\frac{\log a}{\log b}$ (1)

1.1.2 If $x = 9$, then $x^{\frac{1}{2}}=3$ or -3 (1)

1.1.3 $\sec(360^0-40^0)=\sec 40^0$ (1)

1.2 Simplify the following WITHOUT using a calculator:

1.2.1 $\frac{3^2}{2^{-1}}\times\frac{4^0}{27^{\frac{2}{3}}}\div 16^{-2}$ (3)

1.2.2 $\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}-2\sqrt{48}\right)}$ (3)

1.2.3 $\log_c\frac{a}{b}+\log_c\frac{b}{a}$ (2)

1.3 Factorise the following and simplify where possible:

1.3.1 $36-(p+3)^2$ (2)

1.3.2 $x^2-4x+x-4$ (2)

1.3.3 $2p^{16}-\frac{1}{2}$ (2)

1.4 Simplify:

1.4.1 $\frac{x^2-5x+6}{(x-4)^2}\div\frac{x^2+7x+12}{x^2-16}\times\frac{x-4}{x-3}$ (4)

1.4.2 $\frac{4a}{3a+1}+\frac{2a}{1-3a}+\frac{12a^2}{9a^2-1}$ (4)

[25]

QUESTION 2

2.1 Solve for x:

2.1.1 $2^{x+2}+2^{x+1}=192$ (3)

2.1.2 $\sqrt{-3x-5}-5=x$ (3)

2.1.3 $\log_x 125+\log_4 64-\log_3 \frac{1}{3}=\log_2 128$ (3)

2.2 Solve for a by completing the square:
$2a^2-3a-5=0$ (4)

2.3 Make x the subject of the formula:

2.3.1 $M+N=P^x$ (2)

2.3.2 $h=d+\frac{x^2}{d}$ (2)

2.3.3 $\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$ (2)

2.4 Calculate the value of g in the following equation if $t=\frac{1}{2}$; $m=9$ and $f=16$

$$t=\frac{m}{12}\sqrt{\frac{f}{g}}$$ (3)

2.5 The length of a rectangular floor is 3 m longer than its width. The area of the floor is $54\ m^2$. Calculate the length and the width of the floor. (3)
[25]

QUESTION 3

3.1 Sketch the graphs of the following equations in the ANSWER BOOK. Each graph should be on its own system of axes. Calculations need NOT be shown. ALL values at the points of intersection with axes and the coordinates of the turning points (where applicable) MUST be shown:

3.1.1 $xy+4=0$ (Show coordinates of two points through which the graphs pass in each quadrant, that is, 4 points altogether) (2)

3.1.2 $y=-\sqrt{36-x^2}$ (2)

3.2 Solve the following simultaneous equations graphically:
$y = 2x^2$ and $y = 2x$. (5)

3.3 Given: $P(1;7)$ and $Q(3:-1)$ are two points in a Cartesian plane.

Calculate the following:

3.3.1 The length of PQ. Leave the answer in simplified surd form. (2)

3.3.2 The coordinates of M, the midpoint of PQ. (2)

3.3.3 The equation of PQ. Write the answer in gradient-intercept form. (3)

3.3.4 The inclination of line PQ. (2)

3.3.5 The equation of the line perpendicular to PQ which passes through M (3)

3.4 $(b;-\sqrt{2})$ is a point on the circle $x^2 + y^2 = 18$. Determine the values of b. (2)

[23]

QUESTION 4

4.1 Differentiate from first principles:

$f(x) = 3x^2 + 2$ (4)

4.2 Calculate $\frac{d}{dx}\left[2\sqrt{x} + 3k^2 - \frac{2}{x}\right]$ by using the rules of differentiation. Leave the answers in surd form and with positive exponents.

(3)

4.3 Calculate the x-ordinates of the turning points of the following function:

$f(x) = x^3 - 12x^2 + 36x$ (3)

[10]

QUESTION 5

5.1 Calculate the exact value of the following:
(Calculators must NOT be used.)

$$\sqrt{(\cos^2 240° + \cos^2 135°)\sec 300° \tan 225°}$$ (4)

5.2 Make use of trigonometric identities to prove the following:

$$\frac{1+\tan^2 \beta}{\tan \beta . \sec \beta} = \cos ec\beta$$ (4)

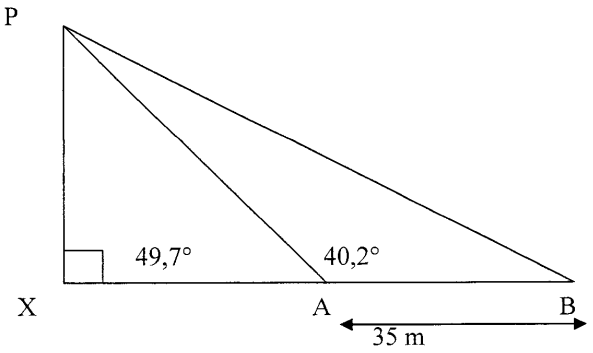
5.3 Calculate the value(s) of θ that will satisfy the equation (with $0° \leq \theta \leq 360°$):

$$2\tan\theta - 4 = 0$$ (3)

5.4 A and B are two beacons 35 m apart. X, A and B are in the same straight line. If $\hat{B} = 40{,}2°$ and $P\hat{A}X = 49{,}7°$, calculate:

5.4.1 PA (2)

5.4.2 PX (2)



5.5 Sketch the graph of the following trigonometric function for $0° \leq x \leq 180°$:

$$y = 3\cos 2x$$ (2)

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TOTAL: 100

MATHEMATICS N3

FORMULA SHEET

Any applicable formula may also be used.

1. Factors

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. Circle

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

2. Logarithms

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \therefore \ln e = 1$$

$$a^{\log_a t} = t \therefore e^{\ln m} = m$$

6. Straight Line

$$y - y_1 = m(x - x_1)$$

Perpendicular: $m_1 \bullet m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination: $\theta = \tan^{-1} m$

7. Differentiation

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Maks/Min

For turning points: $f^1(x) = 0$

8. Trigonometry

$$\sin\theta = \frac{y}{r} = \frac{1}{\cos ec\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \cos ec^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$