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higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1400(E)(N9)T
NOVEMBER 2011

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

9 November (X-Paper)
09:00 – 12:00

ERRATA

QUESTION 1

1.2 If $z = x\sin y - y\sin x$, calculate $\frac{\partial z}{\partial x} - \frac{\partial^2 z}{\partial x \partial y}$

QUESTION 3

Use partial fractions to calculate the following **integrals**:

3.1 $\int \frac{4x^2 - 14x - 10}{(2x+3)^2(2x-1)}$

4.2 Find the general solution of the following:

$$2\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=x^2$$

QUESTION 5

5.2 5.2.1 Sketch the graph of $\frac{x^2}{4}-\frac{y^2}{9}=1$. Show the representative strip/element, perpendicular to the x-axis, that you will use to calculate the volume if the area bounded by the graph and the line $x=4$ is rotated about the x-axis.

5.4 5.4.1 A vertical sluice gate, in the form of a parabola is installed in a dam wall. The top of the sluice gate is 7 m wide and lies in the water level. The sluice gate is 2 m high. Make a neat sketch of the sluice gate and show the **representative** strip/element that you will use to calculate the depth of the centre of pressure on the sluice gate.

Calculate the relation between the variables x and y.

QUESTION 6

6.2 Calculate the surface area of a body generated by revolving the arc represented by $x=2(\theta-\sin\theta)$ and $y=2(1-\cos\theta)$ between $\theta=0$ and $\theta=\frac{\pi}{4}$, about the x-axis.

HINT: $\cos\theta=1-2\sin^2\frac{\theta}{2}$

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This question paper consists of 5 pages and 7-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Questions may be answered in any order, but subsections of questions must be kept together.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If the parametric equations of a function are given as:

QUESTION 2

Determine $\int y \, dx$ if:

2.1 $$y = \frac{\cos^3 3x}{\sqrt[3]{\sin 3x}}$$

2.2 $$y = e^{3x} . \sin 4x$$

2.3 $$y = \frac{1}{\sqrt{-x - x^2}}$$

2.4 $$y = \cot^4 5x$$

2.5 $$y = \tan^{-1} x$$

QUESTION 3

Use partial fractions to calculate the following integravals:

.1 $$\int \frac{4x^2 - 14x - 10}{(2x+1)^2 (2x-1)} dx$$

.2 $$\int \frac{2x - 1}{(x+1)(x^2+4)} dx$$

QUESTION 4

.1 Determine the particular solution of the following:

$$(x+1)\frac{dy}{dx} - y = (x+1)^4 \text{ by } (0;2)$$

.2 Find the general solution of the following:

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $x = y^2$ and $y = x^3$. Sketch the graphs and show the representative strip/element that you will use to calculate the area bounded by the graphs.

5.1.2 Calculate the area described in QUESTION 5.1.1.

5.1.3 Calculate the y-ordinate of the centroïd of the area described in QUESTION 5.1.1.

5.2 5.2.1 Sketch the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$. Show the representative strip/element, perpendicular to the x-axis. that you will use to calculate the volume if the area bounded by the graph and the line $x = 4$ is rotated about the x-axis.

5.2.2 Calculate the volume described in QUESTION 5.2.1.

5.2.3 Calculate the distance of the centre of gravity from the y-axis of the solid generated when the area in QUESTION 5.2.1 is rotated about the x-axis.

5.3 5.3.1 Make a neat sketch of $y = x^2$ and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the curve, the lines $x = 2$, $x = 1$ and $y = 0$ is rotated about the y-axis.

5.3.2 Calculate the magnitude of the volume described in QUESTION 5.3.1.

5.3.3 Calculate the moment of inertia about the y-axis of the solid generated when the area, described in QUESTION 5.3.1, is rotated about the y-axis.

5.4 5.4.1 A vertical sluice gate, in the form of a parabola is installed in a dam wall. The top of the sluice gate is 7 m wide and lies in the water level. The sluice gate is 2 m high. Make a neat sketch of the sluice gate and show the representation strip/element that you will use to calculate the depth of the centre of pressure on the sluice gate.

Calculate the relation between the variables x and y.

5.4.2 Calculate the area moment of the sluice gate about the water level and the depth of the centre of pressure on the sluice gate if the second moment of area about the water level is 8,62 $(\text{units})^4$.

QUESTION 6

6.1 Calculate the arc length of the curve given by the following:

$y^2 = 4x$ between $y = 0$ and $y = 4$

6.2 Calculate the surface area of a body generated by revolving the arc represented by $x = 2\,(\theta - \sin\theta)$ and $y = 2(1 - \cos\theta)$ between $\theta = 0$ and $\theta \frac{\pi}{4}$, about the x-axis.

HINT: $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

[

TOTAL: 1

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\sin x$. 1 1

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \tfrac{1}{2} - \tfrac{1}{2} \cos 2A$

$\cos^2 A = \tfrac{1}{2} + \tfrac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \tfrac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \tfrac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \tfrac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \tfrac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\sin x$ 1 1

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx} (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx} (dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} \cdot \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$- \frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2 (ax)$	-	$-\frac{1}{a} \cot (ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2} \sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$$

Applications of integration

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore\ ye^{\int P dx} = \int Qe^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$