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higher education
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Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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APRIL 2012

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

22 March (X-Paper)
09:00 – 12:00

Calculators may be used.

This question paper consists of 5 pages and a 7-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions MUST be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written out clearly.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks are indicated as percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 Determine the value of:

$$\frac{\partial^2 V}{\partial x \partial y} \text{ if } V = \log_e(2y - 3x)$$ (3)

1.2 Calculate $\frac{d^2 y}{dx^2}$ if $y = \cos 2t$ and $x = 4\sin t$ (3)

[6]

QUESTION 2

Determine $\int y \, dx$ if:

2.1 $y = \tan^3 \frac{x}{2}$ (4)

2.2 $y = \frac{1}{\sqrt{1 - x - x^2}}$ (3)

2.3 $y = \cos^5 3x . \sin^3 3x$ (4)

2.4 $y = x^2 . e^{-x}$ (4)

2.5 $y = \tan^{-1} 6x$ (3)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^3 + 1}{x^2 (x-1)^2} \, dx$ (6)

3.2 $\int \frac{x^3 - 4x^2 + 2x - 1}{x^2 (3x^2 - 1)} \, dx$ (6)

[12]

QUESTION 4

4.1 Find the particular solution of:

$$\sin x \frac{dy}{dx} - y\cos x = 2 \quad \text{if } y = 3 \text{ when } x = \frac{\pi}{4}$$ (6)

4.2 Find the general solution of:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 7y = e^x$$ (6)

[12]

QUESTION 5

5.1 5.1.1 Sketch the graph of $\frac{x^2}{4} + \frac{y^2}{16} = 1$ and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the graph and the x-axis which lies above the x-axis, rotates about the y-axis. (2)

5.1.2 Calculate the magnitude of the volume described in QUESTION 5.1.1 (3)

5.1.3 Calculate the distance of the centre of gravity from the x-axis of the solid generated when the area in QUESTION 5.1.1 is rotated about the y-axis. (4)

5.2 5.2.1 Sketch the graphs of $y = 4x + 3$, $2y + 4x - 6 = 0$ and $x = 1{,}5$ and show the area bounded (enclosed) by the graphs. Show the representative strip/element that you will use to calculate the area bounded by the graphs. (3)

5.2.2 Calculate, by means of integration, the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the area moment about the x-axis and also the distance of the centroid from the x-axis. (5)

5.3 5.3.1 Calculate the points of intersection of $y = x^3$ and $y = x$. Sketch the graphs and show the representative strip/element (parallel to the x-axis) that you will use to calculate the area in the first quadrant, bounded by the graphs, with respect to the x-axis. (3)

5.3.2 Calculate the area described in QUESTION 5.3.1. (3)

5.3.3 Calculate the second moment of area of the area described in QUESTION 5.3.1. (4)

5.4 5.4.1 A vertical weir in the shape of a trapezium is 10 m wide at the top, 6 m wide at the bottom and 2 m high. The top of the weir is 1 m below the water surface.

Make a neat sketch of the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure. Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate the area moment of the weir about the water surface by means of integration. (3)

5.4.3 Calculate the second moment of area of the weir about the water surface as well as the depth of the centre of pressure on the weir. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $x = 2t^2 + 4$ and $y = 2t^3$ between the points t = 0 en t = 3. (6)

6.2 Determine the surface area generated when the arc of the curve $x^2 + y^2 = 25$ rotates about the x-axis for $-5 \leq x \leq 5$. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx} (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx} (dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} \cdot \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \text{ cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\text{cosec } ax$	$-a \text{ cosec } ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

PTO

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

PTO

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2 (ax)$	-	$-\frac{1}{a} \cot(ax) - x + C$

$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$

$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2} \sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$

Applications of integration

AREAS

$A_x = \int_a^b y\,dx \; ; \; A_x = \int_a^b (y_1 - y_2)\, dx$

$A_y = \int_a^b x\,dy \; ; \; A_y = \int_a^b (x_1 - x_2)\, dy$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

PTO

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore\ ye^{\int P dx} = \int Qe^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$