249

higher education
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Department:
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REPUBLIC OF SOUTH AFRICA

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NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

22 March (X-Paper)
09:00 – 12:00

This question paper consists of 5 pages and a 7-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 Determine the value of $\frac{d^2y}{dx^2}$ for the following set of parametric equations:

$x = 3t$ and $y = \ell n\ (1 - t)$ (3)

1.2 A container is expanding along its length at 0,3 m/s and along its breadth at 0,025 m/s, but is contracting along its depth at 0,04 m/s.

Calculate the change in volume if the length is 2,8 m, the breadth 1,5 m and the depth 0,9 m.

HINT: $\Delta V = \frac{\partial V}{\partial l}\Delta l + \frac{\partial V}{\partial b}\Delta b + \frac{\partial V}{\partial h}\Delta h$ (3)

[6]

QUESTION 2

Determine $\int y \ dx$ if:

2.1 $y = \frac{1}{\sqrt{15 + 2x - x^2}}$ (3)

2.2 $y = \tan^4 2x$ (4)

2.3 $y = e^{\frac{1}{2}x} . \sin x$ (5)

2.4 $y = \sin^5 x . \cos^5 x.$ (4)

2.5 $y = x^4 . \ell n\, x$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{2x - 4}{x^2(5x - 2)} \ dx$ (5)

3.2 $\int \frac{15x^3 + 11x^2 + 7x + 5}{(x^2 + 1)(3x + 1)^2} \ dx$ (7)

[12]

QUESTION 4

4.1 Bereken die algemene oplossing van:

$\tan x \frac{dy}{dx} + 2y = x \ . \operatorname{cosec} x$ (5)

4.2 Calculate the particular solution of:

$$2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 12y = e^{3x} \text{ at } (0;1) \text{ and if } \frac{dy}{dx} = 10 \text{ when } x = 0$$

(7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = 3x$ and $y = 7x - x^3$. Sketch the graphs and show the area bounded by the two curves. Show the representative strip/element that you will use to calculate the volume generated when the area bounded by the two curves, in the FIRST quadrant, is rotated about the x–axis. (4)

5.1.2 Calculate the magnitude of the volume described in QUESTION 5.1.1. (4)

5.2 5.2.1 Calculate the points of intersection of $y = 4x^2$ and $y = 5x - 1$. Sketch the curves and show the representative strip/element, PERPENDICULAR to the x–axis that you will use to calculate the area bounded by the two curves. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1, bounded by the two curves. (3)

5.2.3 Calculate the second moment of area of the above-mentioned area about the y-axis and express the answer in terms of the area. (5)

5.3 5.3.1 Sketch the graph of $y = -x^2 + 16$ and show the representative strip/element that you will use to calculate the area bounded by the graph and the lines $y = 0$ and $y = 4$. (2)

5.3.2 Calculate the magnitude of the area described in QUESTION 5.3.1. (4)

5.3.3 Calculate the y-ordinate of the centroïd of the area described in QUESTION 5.3.1. (5)

5.4 5.4.1 A vertical sluice gate in the form of a trapezium is 4 m high. The longest horizontal side is 6 m in length and 2 m below the water surface. The shorter side is 4 m in length and 6 m below the water surface. Make a neat sketch of the sluice gate and calculate the relation between the two variables x and y. (3)

5.4.2 Calculate the area moment of the sluice gate about the water level. (3)

5.4.3 Calculate the second moment of area of the sluice gate about the water level, as well as the depth of the centre of pressure on the sluice gate by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $y = 3x^2 - 4$ between the points $x = 0$ to $x = 2$. (6)

6.2 Calculate the surface area of the solid of revolution generated by rotating the curve described by the parametric equations $x = ct$ and $y = ct^2$ between $t = 1$ and $t = 3$ about the y-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2 ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\,f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore\, ye^{\int P dx} = \int Qe^{\int P dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$