DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

AUGUST 2012

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If $y = \sin 3\theta . \cos 3a$, determine the value of:

1.1.1 $\frac{\partial y}{\partial \theta}$ (1)

1.1.2 $\frac{\partial y}{\partial \alpha}$ (1)

1.1.3 $\frac{\partial^2 y}{\partial \theta^2}$ (1)

1.2 Find the equation of the tangent to the curve given by the parametric equations, $y = \frac{1}{3}e^{2t}$ and $x = 5e^{-2t}$, at the point where $t = 0$. (3)

[6]

QUESTION 2

Determine $\int y \, dx$ if:

2.1 $y = \frac{3}{\cot^4 2x}$ (4)

2.2 $y = e^{\frac{x}{3}} . \cos x$ (5)

2.3 $y = \frac{1}{\sqrt{4 - 8x - 2x^2}}$ (4)

2.4 $y = \sin^4 8x . \cos^3 8x$ (5)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^2 + 2x + 9}{(2x - 3)^3} \, dx$ (6)

3.2 $\int \frac{x^2 - 2x + 2}{(x^2 - x - 1)(x - 1)} \, dx$ (6)

[12]

QUESTION 4

4.1 Calculate the general solution of:

$$\frac{dy}{dx} - (3\tan x)\,y = \sin^6 x$$ (5)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 39 \text{ if } x = 0 \text{ when } y = 2 \text{ and } x = 0 \text{ when } \frac{dy}{dx} = 1.$$ (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $4y = x^2$ and $y^2 = 4x$. Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element that you will use to calculate the volume (use the SHELL METHOD only) generated if the area bounded by the curves rotates about the x-axis. (3)

5.1.2 Use the SHELL METHOD to calculate the volume described in QUESTION 5.1.1 by means of integration. (4)

5.2 5.2.1 Calculate the points of intersection of $y = \frac{16}{x}$ and $y + 2x - 12 = 0$.

Sketch the graphs and show the representative strip/element that you will use to calculate the area in the first quadrant, bounded by the graphs. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the y-ordinate of the centroid of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Make a neat sketch of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and show the representative strip/element that you will use to calculate the volume generated when the area in the first quadrant, bounded by the curve, the y-axis, the x-axis and the line $y = 3$ is rotated about the y-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid obtained when the area in QUESTION 5.3.1 rotates about the y-axis. (4)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the mass. (1)

5.4 5.4.1 A water canal in the form of a parabola is 4 m deep and 4 m wide on the top and full of water. The top of a vertical retaining wall is 1 m below the surface of the water. Sketch the water canal and show the representative strip that you will use to calculate the depth of the centre of pressure on the retaining wall.

Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate the area moment of the retaining wall about the water level by means of integration. (4)

5.4.3 Calculate the second moment of area of the retaining wall around the water level, as well as the depth of the centre of pressure on the retaining wall by means of integration. (5)
[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $y = \frac{1}{2}sin^3\theta$ and $x = \frac{1}{2}\cos^3\theta$ between the points $\theta = 0$ and $\theta = \frac{\pi}{2}$. (6)

6.2 Calculate the surface area generated when the arc of $x^2 = 4y$, between $y = 1$ and $y = 3$ is rotated about the y-axis. (6)
[12]

TOTAL: 100