DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

APRIL 2013

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

CALCULATORS MAY BE USED.

QUESTION 1

1.1 Given: $z = \cos\left(\frac{2x}{y}\right)$. Determine the value of $\frac{\partial^2 z}{\partial x^2}$. (3)

1.2 The dimensions of a cone are r = 25 mm and h = 70 mm. Calculate the approximate increase in the volume of the cone when the radius increases by 5 mm and the height decreases by 10,5 mm.

HINT:

The volume of the cone = $\frac{1}{3}\pi r^2 h$. (3)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \sqrt{6x - 3x^2}$ (4)

2.2 $y = e^{4x}(x^2 - 2x + 3)$ (4)

2.3 $y = \frac{1}{\tan^3 5x}$ (4)

2.4 $y = \sin^3\left(\frac{x}{3}\right).\cos^4\left(\frac{x}{3}\right)$ (4)

2.5 $y = \cos^{-1} x$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{2x^3 + 2x - 1}{(x-1)^3(x+2)}\, dx$ (6)

3.2 $\int \frac{(x-1)}{x^3 + 4x}\, dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$x\frac{dy}{dx} + (1-x)y = e^{3x}, \text{ if } x = 0 \text{ when } y = 1.$$ (6)

4.2 Calculate the general solution of:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = x^2 + 2x - 1$$ (6)

[12]

QUESTION 5

5.1 5.1.1 Make a neat sketch of the curve $y = 3\sin 2x$ and show the area bounded by the curve and the lines $x = 0$ and $x = \frac{\pi}{2}$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded rotates about the y-axis. (2)

5.1.2 Calculate the volume described in QUESTION 5.1.1 by means of integration. (5)

5.2 5.2.1 Calculate the points of intersection of $y = 2x$ and $x^2 = \frac{1}{4}y$. Sketch the graphs and show the representative strip/element (PERPENDICULAR to the x-axis) that you will use to calculate the area bounded by the graphs. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the second moment of area described in QUESTION 5.2.1 with respect to the y-axis and express the answer in terms of the area. (5)

5.3 5.3.1 Make a neat sketch of $4x^2 - 9y^2 = 36$ and show the representative strip/element that you will use to calculate the volume of the solid generated when the area bounded by the curve, $y = 0$ and $y = 4$ is rotated about the y-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid obtained when the area in QUESTION 5.3.1 rotates about the y-axis. (5)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the mass. (1)

5.4 5.4.1 A vertical retaining wall in a water canal is 6 m wide at the top, 5 m wide at the bottom and 2 m high. The top of the retaining wall is 2 m below the surface of the water. Sketch the retaining wall and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the retaining wall.

Calculate the relation between the two variables x and y. (4)

5.4.2 Calculate the area moment of the retaining wall about the water level by means of integration. (3)

5.4.3 Calculate the second moment of area of the retaining wall about the water level, as well as the depth of the centre of pressure on the retaining wall by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve represented by the parametric equations, $y = r\,sin^3 2\theta$ and $x = r\cos^3 2\theta$, over the interval $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$. (6)

6.2 Calculate the surface area generated when the arc of $x^2 + y^2 = 16$, between $y = 4$ and $y = 2$, is rotated about the y-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} ; \sin x = \frac{1}{\operatorname{cosec} x} ; \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx} (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx} (dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$- \frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a} \cot(ax) - x + C$

$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$

$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2} \sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$

Applications of integration

AREAS

$A_x = \int_a^b y\,dx \,;\; A_x = \int_a^b (y_1 - y_2)\, dx$

$A_y = \int_a^b x\,dy \,;\; A_y = \int_a^b (x_1 - x_2)\, dy$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$