higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

16 November (X-Paper)
09:00 – 12:00

This question paper consists of 7 pages and a 5-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Questions may be answered in any order, but subsections of questions MUST be kept together.
4. Show ALL the intermediate steps. Simplify where possible.
5. ALL graph work must be done in the ANSWER BOOK.
6. Questions must be answered in blue or black ink.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.

QUESTION 1

1.1 Determine the values of the following limits:

1.1.1 $\lim_{x \to 0}\left(\frac{e^x - 1}{x.e^x}\right)$

QUESTION 2

2.1 Given:

$$f(x) = \frac{1}{(-3x)^5} = \frac{1}{(-3)^5}.x^{-5}$$

Determine the following:

2.1.1 $f(x+h)$

2.1.2 $f(x+h) - f(x)$

2.1.3 $\dfrac{f(x+h)-f(x)}{h}$

2.1.4 $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

2.2 Prove that if $y = \arctan x$, then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$.

2.3 Determine $\dfrac{dy}{dx}$ in each the following cases:
(Simplification is NOT required)

2.3.1 $y = \dfrac{\tan x}{\ell n(2x)}$

2.3.2 $y = e^{\frac{1}{2}x}.\arccos x$

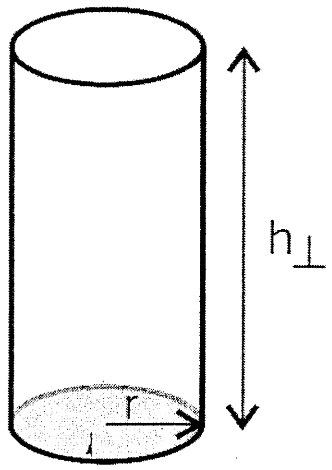
2.3.3 $y = \ell n^4 \left| e^{\ell n x^2} + 1 \right|$

2.4 Calculate $\dfrac{dy}{dx}$ if $y = (\cos x)^{(1+\sin x)}$ with the aid of logarithmic differentiation.

2.5 Determine the slope of the tangent line at the point $(1;1)$ of the curve $e^{y^2}.x = x^4 - 5$.

QUESTION 3

3.1 A liquid falls onto a cylindrical block of ice and solidifies. The volume of the cylindrical block of ice increase at $1000\pi.cm^3.s^{-1}$ and the radius increase at $2{,}5cm.s^{-1}$.



Calculate the rate at which the height changes when the radius is 10 cm and the height is 15 cm.

HINT: $V = \pi r^2 h$

3.2 A particle moves on a horizontal plane with displacement s metres after time t seconds according to,

$s = (3 - \ell nt).t^2$

Determine:

3.2.1 An expression for the velocity of the particle after t seconds.

3.2.2 The acceleration of the particle after 2 seconds.

3.3 Given:

$f(x) = -3x^3 - 3x^2 + 4x + 3$

3.3.1 Determine the coordinates of the turning points of $f(x)$.

3.3.2 Draw up a table of values of x and $f(x)$, with x ranging from $x = -2$ to

3.3.4 Use the table and the graph to estimate a value for the root between $x = -1$ and $x = 0$ of the equation $-3x^3 - 3x^2 + 4x + 3 = 0$ and then use Taylor's/Newton's method to determine a better approximation of this root. (Root correct to THREE decimal figures).

QUESTION 4

.1 Determine the integrals in each of the following cases:

4.1.1 $\int \sin(7\pi x).\cos(3\pi x)dx$

4.1.2 $\int \frac{\cos^2 x - \sin^2 x}{\sqrt{\sin 2x - 4}}dx$

4.1.3 $\int x.\cos(2\pi x).dx$

4.1.4 $\int \frac{1}{\sqrt{11 - x^2}}.dx$

4.1.5 $\int \left(2x^{-\frac{2}{3}}\right).e^{\sqrt[3]{x}}.dx$

4.1.6 $\int \tan^3 x.dx$

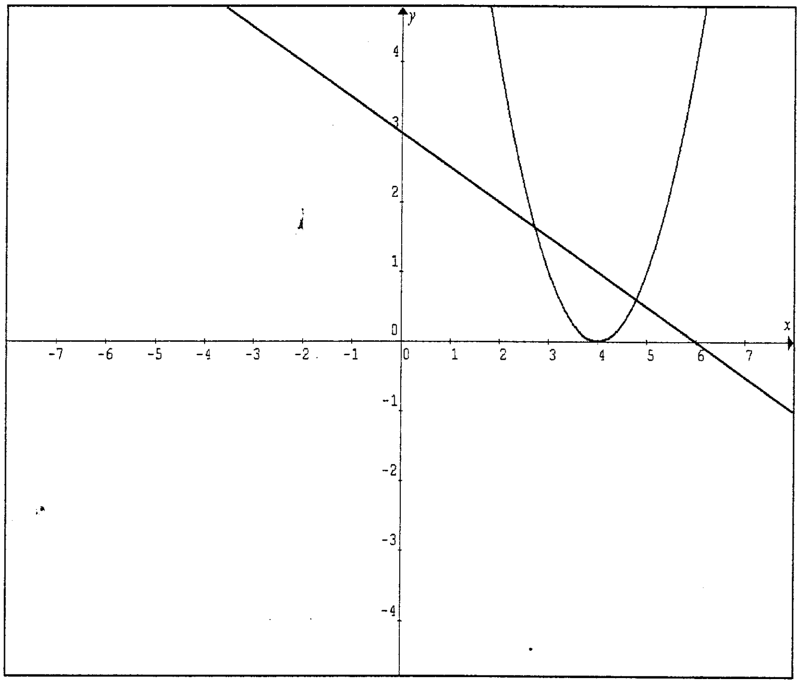
.2 Determine $\int y.dx$ by resolving the integrand into partial fractions:

$$y = \frac{x - 3}{x^2 - 16}$$

QUESTION 5

5.1 Determine $\int_0^\infty e^{-st}.f(t).dt$ if $f(t) = -k$.

5.2 The sketch below shows the area bounded by the curves $y = (x-4)^2$ and $y = -\frac{1}{2}x + 3$. The intersection points between $y = (x-4)^2$ and $y = -\frac{1}{2}x + 3$ are given as $(2{,}719;1{,}640)$ and $(4{,}781;0{,}610)$.



5.2.1 Make a neat sketch to show the area enclosed between the two graphs. (Indicate the points of intersection and the representative strip).

5.2.2 Calculate the magnitude of the area in QUESTION 5.2.1.

5.2.3 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 rotates about the x-axis.

5.3 Determine, from first principles, the second moment of area of a reactangular lamina with respect to a reference axis parallel to one side of the lamina that bisects the

QUESTION 6

Determine the general solution of the following differential equations:

6.1 $$\frac{dy}{dx} = \frac{\pi}{x} - 3$$

6.2 $$x.dy = y.dx - 2.dx$$

6.3 $$\frac{d^2y}{dx^2} + x^{-3} = 4 - \sin x$$

TOTAL:

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} \text{ ; } \sin x = \frac{1}{\text{cosec } x} \text{ ; } \cos x = \frac{1}{\sec x}$$

INOMIAL THEOREM

$$x + h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \ldots$$

IFFERENTIATION

$$= -\frac{f(a)}{f'(a)}$$

$$= a + e$$

RODUCT RULE

$$= u(x) \, . \, v(x)$$

$$= u \, . \, \frac{dv}{dx} + v \, . \, \frac{du}{dx}$$

$$u \cdot v' + v \cdot u'$$

UOTIENT RULE

$$= \frac{u(x)}{v(x)}$$

$$= \frac{v \, . \, \frac{du}{dx} - u \, . \, \frac{dv}{dx}}{v^2}$$

$$\frac{v \cdot u' - u \cdot v'}{v^2}$$

HAIN RULE

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x\ln a}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$\ln(\sec x)+c$
$\cot x$	$-\mathrm{cosec}^2 x$	$\ln(\sin x)+c$
$\sec x$	$\sec x\cdot\tan x$	$\ln[\sec x+\tan x]+c$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x\cdot\cot x$	$\ln(\mathrm{cosec} x-\cot x)+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
	-1	

(x)	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{{}^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{{}^2 - a^2}$	—	$\frac{1}{2a} ln\left(\frac{x - a}{x + a}\right) + c$
$\frac{1}{{}^2 - x^2}$	—	$\frac{1}{2a} ln\left(\frac{a + x}{a - x}\right) + c$

NTEGRATION

$f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\frac{f'(x)}{f(x)}\, dx = \ln\, f(x) + c$

$[f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$\frac{f(x)}{(ax + b)(cx + d)} = \frac{A}{ax + b} + \frac{B}{cx + d}$

$\frac{f(x)}{(x + a)^n} = \frac{A}{(x + a)} + \frac{B}{(x + a)^2} + \frac{C}{(x + a)^3} + \cdots \frac{Z}{(x + a)^n}$

PPLICATIONS OF INTEGRATION

REAS

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; \; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \; ; \; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \; ; \; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$