238

201104T207

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1080(**E**)(M30)T
APRIL 2011

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

30 March (X-Paper)
09:00 – 12:00

This question paper consists of 6 pages and a 5-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps. Simplify where possible.
6. ALL graph work must be done in the ANSWER BOOK.
7. Questions must be answered in blue or black ink.
8. Write neatly and legibly.

QUESTION 1

1.1 Given:

$\log y = \lim\limits_{x \to 2}\left(\dfrac{x^4 - 16}{x^3 - 8}\right)$, calculate the numerical value of:

1.1.1 $\log y$ (3)

1.1.2 y (1)

1.2 Prove that $\lim\limits_{x \to 0}\left(\dfrac{x}{x + \sin x}\right) = \dfrac{1}{2}$ by using L'Hospital's rule. (2)

[6]

QUESTION 2

2.1 Determine $\frac{dy}{dx}$ if $f(x) = \cos x$ by using first principles.
Show ALL intermediate steps. (5)

2.2 Prove that if $y = arc\sec x$, then $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$. (3)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases:

(Simplification is NOT required.)

2.3.1 $y = 6^{\left(x^{-1}.\arctan x\right)}$ (4)

2.3.2 $y = \frac{x.\ell nx}{e^x}$ (4)

2.4 Calculate $\frac{dy}{dx}$ if $y = x^{1-\pi x^2}$ with the aid of logarithmic differentiation. (4)

2.5 Determine $\frac{dy}{dx}$ of the implicit function $y^2x^2 + x - \ell ny = 7$. (5)
[25]

QUESTION 3

3.1 Given: $f(x) = x^3 - x^2 - 4x + 2$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (2)

3.1.2 Draw up a table of values of x and $f(x)$, where x is ranging from $x = -3$ to $x = 3$. (2)

3.1.3 Draw a neat graph of $f(x)$ between these values and show the turning points on it. (2)

3.1.4 Use the table and the graph to estimate a value for the root between $x = -2$ and $x = -1$ of the equation $x^3 - x^2 - 4x + 2 = 0$ and then use Taylor's/Newton's method to determine a better approximation of this root.
(Root correct to THREE decimal figures). (4)

3.2 The difference between two numbers is -12. Calculate the two numbers if the product of the square of one number and the second number is to be a maximum. (5)

3.3 The volume of a sphere increases at $2{,}5\,cm^3.s^{-1}$. Calculate the rate at which the area of the sphere changes when the diameter is $12\,cm$.

HINT: $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$

(5)
[20]

QUESTION 4

4.1 Determine $\int y.dx$ if:

4.1.1 $y = e^{-x}.\sin x$ (4)

4.1.2 $y = \dfrac{2\cos 2x}{\sin x + \cos x}$ (3)

4.1.3 $y = \dfrac{(arc\tan x)^{\frac{1}{4}}}{1+x^2}$ (3)

4.1.4 $y = \dfrac{\sqrt{x}.\cot\sqrt{x}.\cos ec\sqrt{x}}{x}$ (4)

4.1.5 $y = \cos\left(\dfrac{7\pi}{2}x\right).\cos\left(\dfrac{3\pi}{2}x\right)$ (3)

4.2 Determine $\int y.dx$ by resolving the integrand into partial fractions:

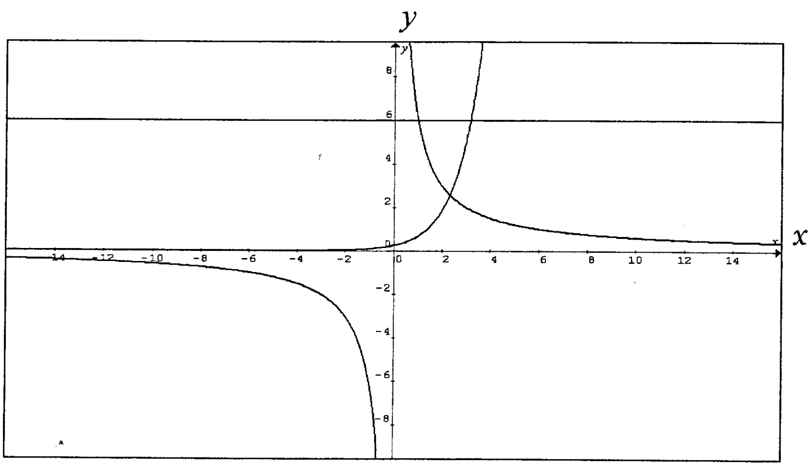
$y = \dfrac{12x}{x^2 - x - 12}$

(5)
[22]

QUESTION 5

5.1 Determine $\int_0^{\infty} e^{-st}.f(t).dt$ if $f(t)=t$. (5)

5.2 The sketch below shows the area bounded by the curves $xy=6$, $y=\frac{1}{4}e^x$ and the line $y=6$. The intersection point between the two curves $xy=6$ and $y=\frac{1}{4}e^x$ is $(2{,}332;2{,}573)$.



5.2.1 Draw the sketch in the ANSWER BOOK and show on the sketch the enclosed area, the representative strip, as well as the lower and upper limits. (3)

5.2.2 Calculate the magnitude of the area in QUESTION 5.2.1.

HINT: $\int \ell n(4y).dy = y.\ell n(4y) - y$ (3)

5.2.3 Calculate the volume generated when the area in QUESTION 5.2.2 rotates about the y-axis.

HINT: $\int (\ell n(4y))^2.dy = y(\ell n(4y))^2 - 2y.\ell n(4y) + 2y$ (4)

5.3 Determine the second moment of mass of a rectangular lamina of mass m about the axis parallel to one side of the lamina. (4)

[19]

QUESTION 6

6.1 Determine the particular solution of $x - \frac{d^2y}{dx^2} = e^x - \frac{1}{x^2}$, given that $\frac{dy}{dx} = -3$, $x = 2$ and $y = 1$. (5)

6.2 Determine the general solution of $\pi.dx - 4y.dy = \frac{1}{y}.dy$. (3)

[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

PTO

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v' + v \cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x\ln a}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$\ln(\sec x)+c$
$\cot x$	$-\mathrm{cosec}^2 x$	$\ln(\sin x)+c$
$\sec x$	$\sec x\cdot\tan x$	$\ln[\sec x+\tan x]+c$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x\cdot\cot x$	$\ln(\mathrm{cosec} x-\cot x)+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\mathrm{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a} ln\left(\frac{x-a}{x+a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a} ln\left(\frac{a+x}{a-x}\right) + c$

INTEGRATION

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$$

$$\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx \; ; \; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x dy \; ; \; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \; ; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA
Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$