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higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1070(**E**)(M28)T
APRIL 2011

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

28 March (X-Paper)
09:00 – 12:00

Calculators may be used.

This question paper consists of 6 pages and a 1-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Answer ALL FIVE questions in full. Show ALL the calculations and intermediary steps. Simplify where possible.
3. Number the answers correctly according to the numbering system used in this question paper.
4. ALL the graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied. Values of intercepts with the system of axes and the turning point(s) MUST be shown on the graph.
5. ALL final answers must be accurately approximated to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must NOT be separated.
7. A formula sheet is attached to this question paper. You are NOT compelled to use the formulae and the list is NOT necessarily complete.
8. Write neatly and legibly.

QUESTION 1

1.1 Given:

$3y - x = 2$

$2y = x$

Solve for x by using Cramer's Rule. (6)

1.2 Given:

$$\begin{vmatrix} 2 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & -3 & 2 \end{vmatrix}$$

Calculate the following:

1.2.1 The determinant of the co-efficients (2)

1.2.2 The co-factor of -1 (1)

1.2.3 The minor of -3 (1)

1.3 Solve for x if:

$e^{x-1} = 7(2^x)$ (3)

1.4 Given:

$10e^{3t} = 51$

Make t the subject of the formula. (3)

1.5 The length of a rectangle is equal to three times the breadth. The area of the rectangle is 48 m^2. Calculate the length and the breadth of the rectangle. (4)

[20]

QUESTION 2

2.1 2.1.1 Sketch the graph of $y = -x^2 + 2x + 3$. (3)

2.1.2 Is the graph of $y = -x^2 + 2x + 3$ discontinuous or not discontinuous? (1)

2.1.3 What is the range of the graph of $y = -x^2 + 2x + 3$? (2)

2.2 Sketch the graph of $xy = -3$. (3)

2.3 Given:

$Z = -2 - j5$

2.3.1 Find $\overline{Z}$ (1)

2.3.2 Convert Z into polar form. Show ALL steps. θ may only be positive. (3)

2.3.3 Represent Z and ALL calculated values in QUESTION 2.3.2 on the Argand diagram. (3)

2.4 Solve for x if:

$2x^2 - 6x + 5 = 0$ (4)
[20]

QUESTION 3

3.1 If $\sin A = \frac{1}{2}$ and $\sin B = \frac{1}{\sqrt{2}}$, and both A and B are acute angles, calculate, WITHOUT using a calculator, the value of $\cos(A - B)$. (4)

3.2 Prove that:

$$\frac{\sin 2x}{\cos 2x + 1} = -\cot x$$ (4)

3.3 Solve for x if:

$4\sin^2 x - 5\cos x = 2;\ 0° \le x \le 360°$ (6)

3.4 Simplify:

$$\frac{2\cot x}{1 + \cot^2 x}$$ (4)

3.5 Derive a formula for $\sin 2A$ (2)
[20]

QUESTION 4

4.1 Given: $y = \frac{x^5}{2^x}$

Differentiate by use of a quotient rule. (4)

4.2 Differentiate $y = 4x^2$ by use of first principles. (4)

4.3 Differentiate with respect to x:

$y = -\sqrt{3}\cos x + 2^{2x} - 4\ell nx - x^{\frac{2}{3}}$ (4)

4.4 Given:

$$y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 7$$

Calculate, with the aid of differentiation; the co-ordinates of the maximum and the minimum turning points. (5)

4.5 Given:

$y = x^3 - 6x^2 + 9x + 12$

Solve for the values of x if $\frac{dy}{dx} = 0$ (3)

[20]

QUSTION 5

5.1 5.1.1 Sketch and indicate the area enclosed by the graph of $y = 4\cos x$ with $x = 0°$ and $x = 90°$. Also indicate the representative strip that will be used to calculate the area enclosed. (3)

5.1.2 Calculate, using Integration, the value of the area indicated in QUESTION 5.1.1. (4)

5.2 Integrate with respect to x:

$$\int \left(\sin 5x + \sec^2 x - 3^x + 9e^{-3x} + 4x^{\frac{1}{4}} \right) dx$$ (6)

5.3 Determine:

$$\int_0^1 2^{-3x}\,dx$$ (4)

5.4 Simplify:

$$\int (4\cos 4x + 3)\,dx$$ (3)

[20]

TOTAL: 100

MATHEMATICS N4

FORMULA SHEET

NEW SYLLABUS

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ell n\, x = \log_e x$

$(r\,|\underline{\theta})^n = r^n\,|\underline{n\theta}$ $\quad a + bj = c + dj \Leftrightarrow a = c$ and $b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \operatorname{cosec}^2 x$
$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
ka^x	$ka^x \ell na$
$k\ell nx$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$y = u(x) \cdot v(x)$

$$\Rightarrow \frac{dy}{dx} = u(x)v'(x) + u'(x)\,v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x} dx = a\ell n\, x + c$$

$$\int ka^x dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y\, dx$$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$\int \tan x\, dx = \ell n \sec x + c$

$\int \sec x\, dx = \ell n (\sec x + \tan x) + c$