# DEPARTMENT OF HIGHER EDUCATION AND TRAINING
# REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

April 2013

**INSTRUCTIONS AND INFORMATION**

1. Answer ALL the questions.
2. Answer ALL FIVE questions in full. Show ALL the calculations and intermediary steps. Simplify where possible.
3. ALL the graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied. The values of intercepts with the system of axes and the turning point(s) must be shown on the graph.
4. ALL final answers must be accurately approximated to THREE decimal places.
5. Questions may be answered in any order, but subsections of questions must NOT be separated.
6. A FORMULA SHEET is attached to this question paper. You are NOT compelled to use the formulae and the list is NOT necessarily complete.
7. Write neatly and legibly.

CALCULATORS MAY BE USED.

 

**QUESTION 1**

1.1 1.1.1 Sketch the graph of $\frac{x^2}{72} - \frac{y^2}{32} = \frac{1}{2}$. (3)

1.1.2 What is the domain of the graph in QUESTION 1.1.1? (2)

1.1.3 Is the graph in QUESTION 1.1.1 symmetrical about both axes? (1)

1.1.4 Does the graph in QUESTION 1.1.1 represent a function or a relation? (1)

1.2 Sketch the graph of $x = -\sqrt{20 - y^2}$. (3)

1.3 Solve for $x$ if:

$x + y + yj + 2j = 8j + 7$ (4)

1.4 Given:

$Z_1 = 1 - j1{,}732$
$Z_2 = -6{,}928 - j4$

1.4.1 Convert $Z_1$ and $Z_2$ into polar form. The arguments may only be positive. (2)

1.4.2 Calculate $\frac{Z_1}{Z_2}$ and leave the answer in $a + jb$ form. (4)
**[20]**

 

## QUESTION 2

2.1 Given:

$$2x - 2y - Z = 3$$
$$4x + 5y - 2Z = -3$$
$$3x + 4y - 3Z = -7$$

2.1.1 Solve for the value of $Z$ by using Cramer's rule. (8)

2.1.2 Determine the value of the cofactor of –3. (2)

2.2 Given:

$$t = \frac{w}{g}\ell og\left(1 + \frac{y}{Z}\right)$$

Make $y$ the subject of the formula. (4)

2.3 Solve for $x$ if:

$$2^{5x-2} = 5^{3x+1}$$ (3)

2.4 The sum of two numbers is 10. The difference of their squares is 50. Calculate the TWO numbers. (3)

**[20]**

## QUESTION 3

3.1 Prove that:

$$\frac{Sin2A - Cos2A + 1}{Sin2A + Cos2A + 1} = \tan A$$ (5)

3.2 Solve for $A$ if:

$$2\cos^2 A = -CosA;\ 0^0 \le A \le 360^0$$ (4)

3.3 3.3.1 Derive a formula for $Cos2A$. (2)

3.3.2 Use the derived formula in QUESTION 3.3.1 to determine the value of $Cos120^0$ without the use of a calculator. (3)

3.4 If $\sin A = 0{,}6$ and $\sin B = 0{,}4$; and both $A$ and $B$ are acute angles, determine the value of $Cos(A + B)$ without the use of a calculator. (3)

3.5 Prove that:

$$\sin(180^0 + A) = -\sin A$$ (3)

**[20]**

 

## QUESTION 4

4.1 Determine:

$$\lim_{x \to 2} \left( \frac{x-2}{x^3-8} \right)$$ (4)

4.2 Given:

$y = e^{3x}.\cos 3x$. Differentiate by using the product rule. (4)

4.3 Differentiate the following with respect to $x$:

$y = 2.3^{5x} - 4\cos 2x - 5\ell nx - 3x^{-1}$ (4)

4.4 Given:

$y = x^3 - 11x^2 + 32x - 28$

Determine, using differentiation, the maximum and the minimum turning points. Also determine the point of inflection by using the second derivative. (8)

**[20]**

## QUESTION 5

5.1 Integrate the following in terms of $x$:

$$\int \left(3\sin 3x - e^{-4x} - 3x^{-4} + 2.3^{5x} - \operatorname{cosec}^2 x + 2\right) dx$$ (7)

5.2 Determine: $\int_1^2 2^{-5x} dx$ (3)

5.3 5.3.1 Sketch and indicate the area enclosed by the graph of $y = 4\cos x$, the $x-axis$, $x = 0$ and $x = \Pi/2$. Also, indicate the representative strip to be used to calculate the area of the enclosed area. (3)

5.3.2 Calculate, using integration, the value of the enclosed area indicated in QUESTION 5.3.1. (4)

5.4 Simplify:

$$\int \frac{1}{\operatorname{cosec} x} dx$$ (3)

**[20]**

**TOTAL: 100**

## FORMULA SHEET

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ell n\, x = \log_e x$

$(r\,|\,\underline{\theta}\,)^n = r^n\,|\,\underline{n\theta} \quad a + bj = c + dj \Leftrightarrow a = c \text{ and } b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \operatorname{cosec}^2 x$
$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

| $y$ | $\frac{dy}{dx}$ |
|---|---|
| $ax^n$ | $nax^{n-1}$ |
| $ka^x$ | $ka^x \ell na$ |
| $k\ell nx$ | $\frac{k}{x}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$y = u(x) \cdot v(x)$

$$\Rightarrow \frac{dy}{dx} = u(x)v'(x) + u'(x)\,v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x}\,dx = a\ell n\, x + c$$

$$\int ka^x\,dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y\,dx$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \tan x\,dx = \ell n \sec x + c$$

$$\int \sec x\,dx = \ell n\,(\sec x + \tan x) + c$$