MATHEMATICS N3 TUTORIAL NOTES : PREPARED BY R.C NEMUDZIVHADI

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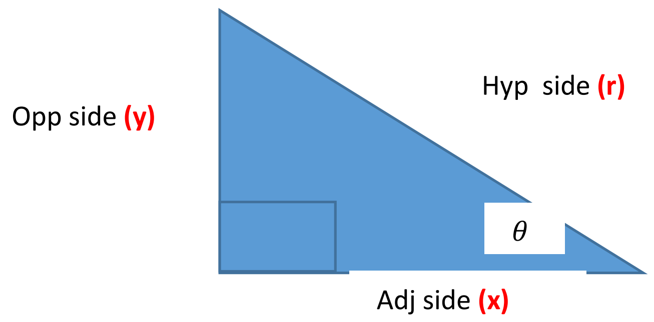
TRIGONOMETRY: $(\pm 26\, marks)$

Students must able to understand the following:

1. Define trig ratios and its reciprocals and theorem of Pythagoras
2. Applying the rules and identities
3. Reduction formula
4. Negative angles
5. Co- ratios
6. Special angles
7. General solutions
8. Solving
9. Sketching all trig functions (sin,cos & tan)
10. Checking all the parameters of the graphs, i.e TP, amplitude, max, min, range & period.
11. Calculate area (height and distance)

1. **Defining trig ratio** (only done from right angle triangle) $\pm 5\, mark$



Basic definitions	
Sin θ = $\frac{opp}{hyp} = \frac{y}{r}$	cosec θ = $\frac{r}{y}$
cos θ = $\frac{adj}{hyp} = \frac{x}{r}$	Sec = $\frac{hpy}{adj} = \frac{r}{x}$
tan θ = $\frac{opp}{adj} = \frac{y}{x}$	cot θ = $\frac{adj}{opp} = \frac{x}{y}$

Example 1.

If 5sin θ = -3, find the value of the following trigonometric ratios

tan θ . sec θ . cosec θ

tips to solve the above problem

step one: draw cartesain plain

step two: name axis in terms of y and x axis

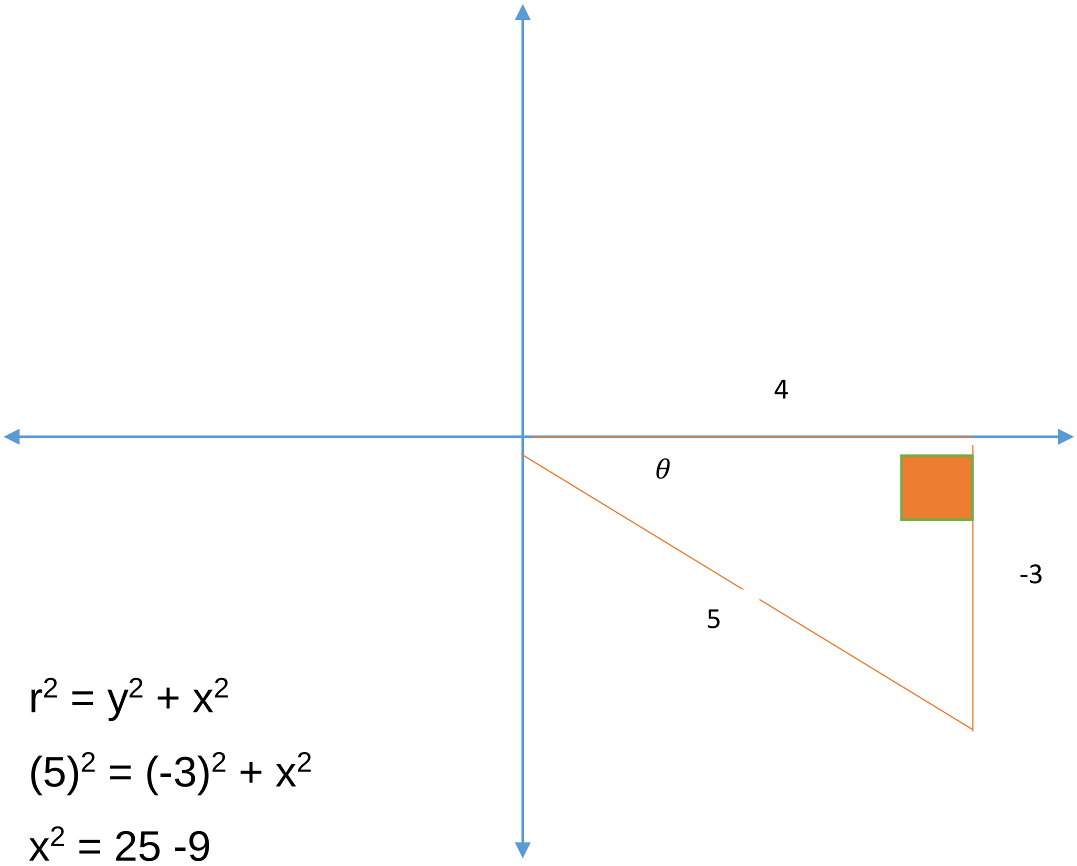
step three: make sin θ the subject of the formula

step four: then check the suitable quadrant for the values (opp = y = -3, hypotenuse = r= 5)

note that hypotenuse side is always positive.

Step five : apply theorem of Pythagoras to calculate the unknow side

Step six: solve the problem using trig definition for each ratio



$r^2 = y^2 + x^2$

$(5)^2 = (-3)^2 + x^2$

$x^2 = 25 - 9$

$x^2 = 16$

$x = \sqrt{16}$

$x = 4$

now use information from the diagram to solve the problem not forgetting definitions of each trig ratio

tan θ . sec θ . cosec θ

$= \frac{opp}{adj} + \frac{adj}{hyp} \times \frac{hyp}{opp}$,

not important during exam, remainder purpose

$= \frac{-3}{4} \times \frac{4}{5} \times \frac{5}{-3}$,

calculator work, make sure you insert every term as it is in your calculator

= 1

2. Applying the rules and identities $(\pm\ 6 \text{ marks})$

1.1. Trigonometric identities

1.1.1. $\tan x = \frac{\sin x}{\cos x}$

1.1.2. $\cos^2 x + \sin^2 x = 1$

1.1.3. $1 + \cot^2 x = \text{cosec}^2 x$

1.1.4. $\tan^2 x + 1 = \sec^2 x$

note that (2.1.3) = $\frac{(2.1.2)}{sin^2\ x}$ and (2.1.4) = $\frac{(2.1.2)}{cos^2\ x}$

<u>Example:</u>

Prove the following identities:

$(\boldsymbol{\sin\theta - \cos\theta})^{\mathbf{2}} = \mathbf{1 - 2\sin\theta\ .\cos\theta}$

Step 1: check longest side or complicated side to work with in order to prove the other side(now from these example work with LHS and multiply out the bracket.

LHS = $(\sin\theta - \cos\theta)^{2}$

$= sin^2\ \theta - sin\theta.\cos\theta - sin\theta.cos\theta + cos^2\theta$

Step 2: add like terms

LHS = $sin^2\,\theta - 2sin\theta cos\theta + \; cos^2\theta$

Step 3: group $sin^2\,\theta + cos^2\theta$, which equals 1

LHS = $sin^2\,\theta + cos^2\theta$ - $2sin\theta cos\theta$

= 1 - $2sin\theta cos\theta$

LHS = RHS

<u>ACTIVITY : Prove the following identity</u>

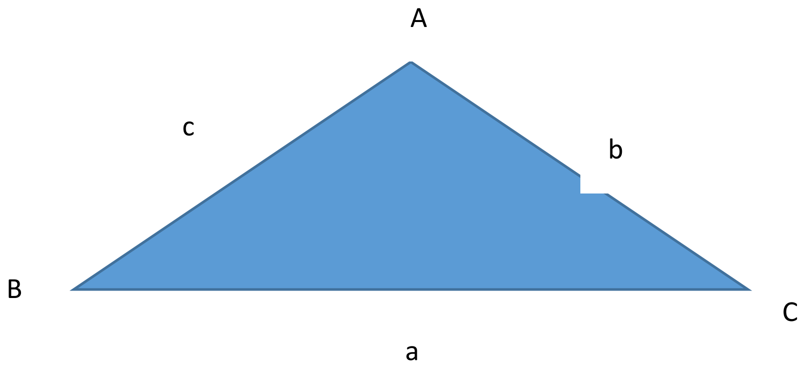
1. $\cos\theta\sin\theta = \frac{\tan\theta}{1+\,tan^2\theta}$

2. $\frac{2sin^2x}{2\tan x - 2sinx\,.cosx} = \frac{\cos x}{\sin x}$

3. $\frac{\frac{\cos x\,.tan^2x}{1}}{\cos x+1} = 1 - \cos x$

The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



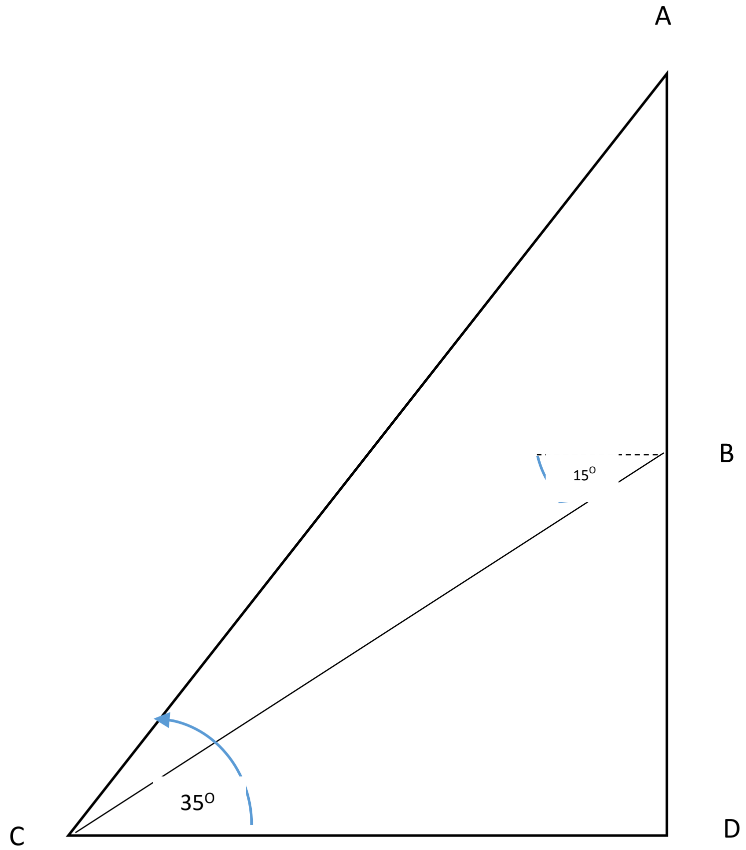
The cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

EXAMPLE



A man, B, stands on the observation deck of a lighthouse. The angle of depression of a child, C, on the rocks below is 15^o. The foot of the tower, D and the child, C, are in the same horizontal plane. The child, C, notices, the top of the lightining conductor, A, at an angle of elevation of 35^o. The vertical distance from the man, B, to the top of lightning conductor, A is 22m i.e. AB = 22m.

Calculate

1. the distance from the child, C, to the man B, on the observation deck i.e. CB

Step 1: redraw the triangle that you are going to work with and produce other triangles where possible. know the theorem of angles, sum of angles in a different triangles

In $\Delta\, ABC$: $A\hat{B}F = 90^{O}$ (corresp $\angle s$)

$\therefore A\hat{B}C = 105^{O}$

And $\hat{C} = 20^{O}$

$\therefore \hat{A} = 180^{0} - (20^{O} + 105^{O})$ (sum of $\angle s\ in\ a$ Δ)

= 55^{o}

step 2: apply correct rule, for sine check if there is given angle and its side and the other with one known value it can be either side or angle.

For cosine rule they will give you one value for eache, i.e. side, side angle.

$\frac{CB}{\sin A} = \frac{AB}{\sin C}$ (CHECK, we know angle A, side AB is also given and $\hat{C}$ $is\ being\ calculated$

STEP 3: substitute the known value from the above rule

$$\frac{CB}{\sin 55^{o}} = \frac{22}{\sin 20^{o}}$$

Step 4 : make unknown the subject of the formula (*CB*)

Multiply both side by sin 55^{o}

CB = $\frac{22}{\sin 20^{o}} \times \sin 55^{o}$

Step 5: use a calculator to find the value *(hint make sure your calculator is in degree i.e. deg)*

CB = 52,7 m

2. how **far** the child, C, is from the foot of the tower D i.e. CD

NB: KEY WORDS are very important like how far, size etc

here we have to check the distance between C and D

step 1 : check triangle with point C and D

ΔBCD

Step 2 : check what is needed to solve the problem is not always the rules even the basic definitions is useful to solve some of the problems

$in\ \Delta BCD : \frac{CD}{52.7} = \cos 15^o$

Step 3 : Make CD the subject of the formula

Multiply both side by 52.7

CD = $52.7 \times \cos 15^o$

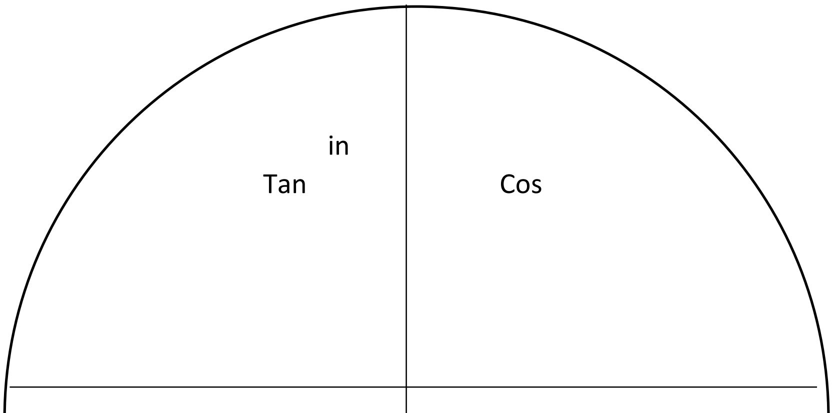
Step 4: use a calculator to find the value

CD = 50,9 m

3, 4 & 6 REDUCTION FORMULAE , NEGATIVE ANGLES & special angles $(\pm 6\ marks)$

Use the diagram the reduce functions of angles greater than 90^o to functions of acute angles. Here are some examples:

CAST RULE (reduction formulae)



HINT:

From 1st quadrant : all ratios are positive

2nd quadrant : only sin and its reciprocal is positive

3rd quadrant : only tan and its reciprocal is positive

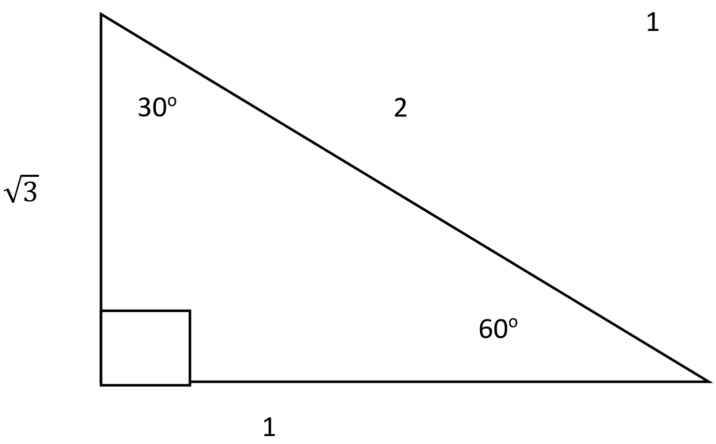
4th quadrant : only cos and its reciprocal is positive

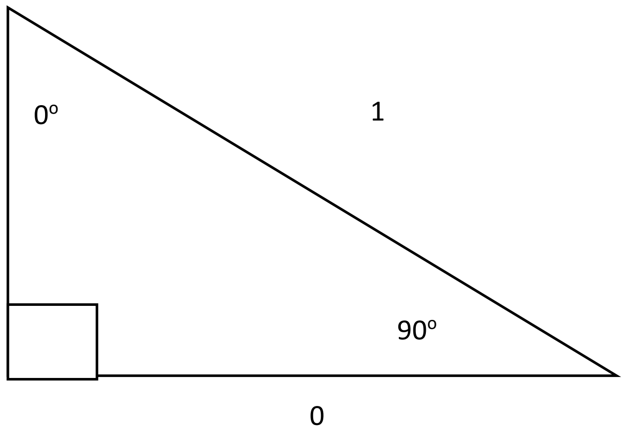
4. Negative angles

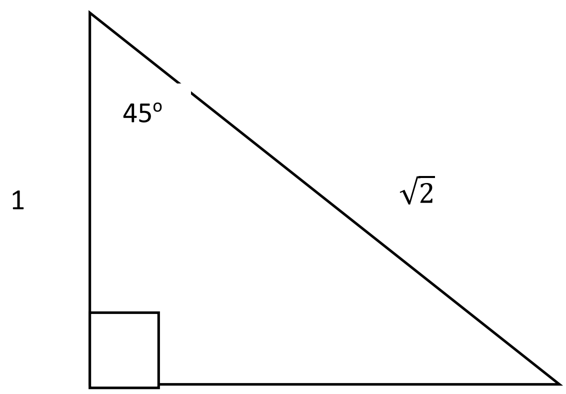
sin (-x) = - sin x

cos (-x) = + cos x

6. special angles







45°

EXAMPLE

1

Simplify the following without using a calculator:

Cos 180° . $\tan^2$150° + sin 300°. cos0° . tan 210°

Step 1: reduce the ratios in terms of 180°$\pm$ and 360° $\pm$

Cos 180° . $\tan^2$150° + sin 300°. cos0° . tan 210°

Cos (180° + 0°) . $\tan^2$ (180° - 30°) + sin (360° - 60°). cos0° . tan (180° + 30°)

Step 2: revisit the CAST rule (remember first angle inside the brackets is your reference angle is telling you where you suppose to start i.e 180° or 360° , the sign between is telling you where to go i.e. clockwise direction (-) or anticlockwise (+), noting the sign, ratio and other remaining angle is the answer.

(-cos0°). (- $\tan^2$30°) + (-sin 60°) .(cos0°). (tan 30°)

Step 3: from these step check if you suppose to use special angles or identities

In our case now we need special angles and here please you define ratio with its definition but respecting given angle.

$(-1)\ (-\frac{1}{\sqrt{3}})^2 + (-\frac{\sqrt{3}}{2}).\ (1).(\frac{1}{\sqrt{3}})$

Step 4: now use a calculator to find the value.

$-\frac{1}{3} - \frac{1}{2}$

$= -\frac{5}{6}$

ACTIVITY

1. $\frac{\sin 130^0 .\tan(-240)^0 .\cos 540^0}{\cos 570^0 .\sin(-300)^0 .\cos 320^0}$

Co- ratios

$90^0 \pm$

When you apply (90^0 + θ) to a function, you will get its co-function. For example, sin θ will change to cos θ and cos θ will change to sin θ

So, sin (90^0 + θ) = cos θ

But cos(90^0 + θ) = - sin $\theta, because$ 90^0 + θ $will\ be\ in\ the\ second\ quadrant\ where \cos\ will\ be\ negative$

The area rule $(\pm\ 6\ marks)$

in any Δ PQR: Area Δ PQR = $\frac{1}{2}pq\ sin\hat{R}$ or

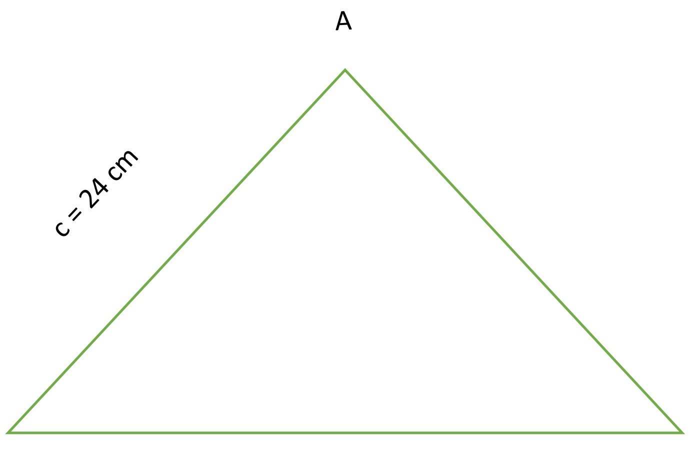
$= \frac{1}{2}pr\ sin\hat{Q}$ or

$= \frac{1}{2}qr\ sin\hat{P}$

EXAMPLE

Determine the area of ΔABC if $\hat{B}$ = 75^0 , a = 16 cm and c = 24 cm

Step 1. Draw the triangle and name it in terms of given dimensions



75^0

B C

a = 16 cm

Step 2: choose the formula you are going to use,

Area Δ ABC = $\frac{1}{2}ac\ sin\hat{B}$

Step 3 : then substitute the values into the formula.

$= \frac{1}{2}(16)(24)\sin 75^0$

Step 4 : then use a calculator to find the area

= 185.5 cm^2

NB: in most cases you can apply the area, sine and cosine rule from same problem.

General solution $(\pm\ 4\ marks)$

Here you will be solving any angle, it can be either liner problem or quadratic.

Hints:

- **Don't solve any expression with different trig ratio(** make sure you are working with one ratio at time i.e expression for sin only no mix up,
- **In case of different ratios, make sure you derive one another with recpect of identity or otherwise before you solve the expression**
- **Check the restrictions and reference angle of that ratio**

Example

Solve for θ, if:

$\sin^2\theta + 2\cos\theta = 4 \quad \theta \in (0^0;\ 360^0)$

Step 1 : make sure you are working with one ratio, if not rewrite the other ratio in the form of other ratio

i.e.$\sin^2\theta = 1 - \cos^2\theta$

step 2: substitute $\sin^2\theta$ with $1 - \cos^2\theta$

step 3: rewrite expression in high degree order

$-\cos^2\theta + 2\cos\theta + 1 - 4 = 0$

Step 4 : add like terms together and multiply the whole expression by (-)

$\cos^2\theta - 2\cos\theta + 3 = 0$

step 5 : check the kind of expression you got and then factorize or apply quadratic formula

$(\cos\theta - 3)(\cos\theta + 1) = 0$

Step 6: equate each factor to zero

$\cos\theta - 3 = 0$ or $\cos\theta + 1 = 0$

step 7 : make θ the subject of the formula

$\theta = cos^{-1}(3)$ or $\theta = cos^{-1}(-1)$

note that your calculator is in degrees

θ Type equation here.= no sol or $\theta = 180^0$

step 8 : from these step we check quadrants where cos is positive (1st and 4th)

$\theta \in (180^0, 360^0 + 180^0)$

$\therefore \theta \in (180^0, 540^0)$

Trig functions

- The sine and cosine functions have several distinct characteristics.

1. Period is 2π
2. the domain of each function is (- ∞ ; ∞)
3. the range [-1 ; 1]
4. the graph of y – sin x is symmetrical about the origin, because it is an odd function.
5. it has a maximum and minimum value
6. amplitude

Hint : check the domain given before you sketch the graph i.e (0^0 ; 180^0), from this case they are telling you that you have to start sketch your graph from 0^0 and end at 180^0

- Is better to step by 45^0
- Use a table method both manually or from your calculator to find the value of y

Casio : press start – select table – insert function (for special key keys please be vigilant on what to press before you find special key) – start from 0 – end at 180 – step by 45 then equal sign

- Then recopy your table on your book, then sketch your graph your calculated co-ordinates.

Enjoy your trig chapter (wish you the best for your coming exams)

GOD BLESS!!